Accountability and incumbency (dis)advantage [1]

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Abstract

This paper analyses the problem that an incumbent faces during the legislature when deciding how to react to citizen mandates such as the outcome of referenda or popular initiatives. We argue that these mandates constitute a potential source of incumbency (dis)advantage when citizens factor into their evaluation of the incumbent his reaction to these proposals. We characterize conditions under which incumbents use these policy decisions in their advantage. This is more likely to be the case the higher the importance citizens award to their mandate, the smaller the disalignment between the incumbent and the citizens on the issue their mandate refers to, and the more office motivated the incumbent is. Otherwise, the incumbent chooses to ignore the citizens' proposal at the risk of losing reelection. Finally, we apply our findings to the experience with participatory democracy in Brazil and to the responsiveness of politicians to popular initiatives in US states.

Keywords: Incumbency advantage, Referenda, Popular initiatives, Elections.

JEL-codes: D7, H1.

1 Introduction

The incumbency advantage is a well documented phenomenon, according to which an incumbent politician is more likely to be reelected than a challenger candidate. Empirical studies, such as Gelman and King (1990) and Lee (2008), provide strong evidence in favor of the existence of such advantage in the US House.[1] Ansolabehere and Snyder (2002) find empirical support for the incumbency advantage hypothesis in US state executives also. These authors argue that incumbency advantage does not originate in the strategic policy choices made by incumbents but in their innate characteristics.[2]

The present paper provides an explanation to the phenomenon of incumbency advantage that focuses on a strategic mechanism. During his term in office, the incumbent must often implement some policies in response to new or emerging common value issues. These policy choices may be costly in terms of chances of reelection if they are unpopular among voters. The incumbent is thus facing implicit restrictions on the policies that he can implement if he wants to remain in office. We analyze the extent to which incumbents are willing to make policy sacrifices during their time in office in order to enhance their electoral prospects.

Although our model is more general, we have in mind a specific type of policy choices as the origin of this potential incumbency disadvantage: incumbents' response to the outcomes of forms of citizen direct political participation. The outcomes of processes like referenda, citizen initiatives or popular assemblies may constrain incumbents because their reaction to these proposals factors into voters' evaluation of the incumbent's performance. They can create an electoral disadvantage compared to the case of an incumbent who does not face such mandates. But also compared to the challenger, whose position does not require him to make any policy choice prior to the electoral campaign and whose chance of winning could improve if the incumbent is not ready to compromise.

[1] Other studies assume that incumbents have better ways to influence voters' decisions than challengers through mechanisms such as redistricting (Levitt and Wolfram 1997, Cox and Katz 2002), seniority (McKelvey and Reizman 1992), informational advantages (Krehbiel and Wright 1983), better access to campaign resources (Goodliffe 2001, Jacobson 2001), legislative irresponsibility (Fiorina 1989) or pork barrel politics (Cain, Ferejohn, and Fiorina 1987, Ansolabehere, Snyder, and Stewart 2000).

[2] In this line, Bevia and Llavador (2009) show that only good quality incumbents may enjoy an advantage. Ashworth and de Mesquita (2008) show that incumbents' quality and ability are higher on average than the challengers'. Gowrisankaran, Mitchell, and Moro (2008) find that incumbents face weaker challengers than candidates who face open seats and Stone, Maisel, and Maestas (2004) find that incumbents' personal qualities deter strong challengers from running for office.

We build a formal model of electoral competition with two candidates, two issues and three stages. The first stage takes place during the legislature, that is, in between two elections. In this stage, the incumbent faces an exogenously given policy proposal on a certain issue. This proposal comes from the constituency and it represents a significant part of the population. That is why we call this issue the *popular issue.* This proposal is non-binding so the incumbent has full discretion over the policy implemented in this issue. The implementation of his choice on the popular issue takes place during the legislature and before the next electoral campaign. The second stage is the electoral campaign stage. In this stage, both candidates announce simultaneously their policy platforms on the *electoral issue,* which is assumed to be different from the popular issue. The electoral issue is defined in the same way as in most models of electoral competition; the candidates' choices over this issue represent their campaign promises over the policies that they will implement in case they win the election. Finally, in the third stage, voters vote for their most preferred candidate taking into account all the information that they have at that point from each of the candidates.

The model presents two types of asymmetries. First, voters evaluate the two candidates differently. We assume that voters use all the information they have available at the time of the election in order to decide who to vote for. Thus, voters evaluate both candidates according to their campaign promises because this information is available from both candidates at the time voters cast their vote. At the time of the election, voters also have information about the policy choice that the incumbent made during his time in office. It is plausible to assume that voters remember specially well those policy choices related to popular demands. These choices should be easier to recall the more intense the popular demand process had been. Of course, whenever there is a strong popular mandate, voters should remember not only the actions taken by the incumbent, but also those taken by the challenger. However, the asymmetric position that the two candidates held during the legislature makes the choices of the incumbent much more relevant than those of the challenger. In particular, the incumbent, being at the head of the executive body, has the power to produce a real policy reaction to any popular demand, and thus the voters can evaluate accordingly his action (or inaction) when facing an intense popular initiative. On the other hand, the power that the challenger has as policy decision maker is much less effective. The challenger can, of course, take a stance favorable or against the popular initiative. But the implementation of his proposal depends, not only on his deeds but on the overall position of the corresponding legislative body. Therefore, the implications of the policy choices of the incumbent during the legislature, as well as their credibility, are much more relevant for the voters than those of the challenger. And voters are more likely to remember and use them at the time of the election. For this reason we model the roles of incumbent and challenger asymmetrically. We construct a reduced form model that highlights the role of the incumbent in the policy choices made during the legislature, and assumes that the role of the challenger during the legislature is negligible. We assume that when voters evaluate the incumbent, they take into account his choice on the popular issue during the legislature in addition to his promises during the electoral campaign, and when voters evaluate the challenger they only take into account his proposed policy on the electoral issue. Similarly, we assume that the incumbent's payoff function depends on his choices on both issues, and therefore when choosing an optimal strategy he has to solve a trade-off between them. However, the challenger optimization problem is simpler, because his position on the popular issue cannot affect the policy implemented on that issue (it is decided by the incumbent alone). This is a stark assumption that allows us to solve the model in a simple and intuitive way and to obtain a closed form solution.

The second asymmetry refers to the two issues. This asymmetry arises because voters realize that they cannot give the same consideration to a policy implemented during the legislature and to an electoral promise, which will be implemented only if the candidate making that promise is elected. The way we model the evaluation made by citizens of the candidates performance and electoral promises can be interpreted as a combination of retrospective voting and prospective voting. On the one hand, voters use retrospective voting to evaluate the performance of the incumbent with respect to the policy he implemented on the popular issue during his time in office. On the other hand, voters use prospective voting to evaluate the campaign promises that both candidates announce during the electoral campaign on the electoral issue. We introduce this asymmetry across issues by assigning different weights to the evaluation of the incumbent in each one of the issues. Furthermore, we assume that citizens use different reference points when evaluating candidates' policy choices. Specifically, we assume that citizens evaluate candidates on the electoral issue by comparing their own preferred policy with each candidate's political platform. However, when citizens' evaluate the performance of the incumbent on the popular issue they compare the incumbent policy choice with the proposal the incumbent received from the constituency, that is, we assume the popular issue to be a common value issue. This assumption is justified because the policy proposal that the incumbent receives is the outcome of a participatory process; a significant part of the population solved their conflict of interests on this issue and united in defending a particular policy. Thus, it is natural that citizens evaluate the incumbent's performance on this issue according to the extent of his compliance to the policy proposal they submitted to him.

In equilibrium, the policy choices of the incumbent in both issues reflect the trade-off he faces between his own policy preferences and his chances of reelection. The incumbent anticipates the strategic choices of the challenger on the electoral issue when he decides about the policy he implements on the popular issue. We find that for all parameter values, the incumbent has a strategy that allows him to be reelected. The question is whether this winning strategy is always optimal for the incumbent. And the answer is no. There are some instances where the incumbent prefers to forgo reelection because the optimal strategy that guarantees his victory is too costly in terms of policy compromises. In those cases, the incumbent implements his ideal policy in the popular issue. For this to happen three conditions must hold: (1) the incumbent must be sufficiently policy-motivated; (2) there must exist a severe disagreement between voters and the incumbent over the popular issue; and (3) voters must assign a high weight to the electoral issue when evaluating the incumbent. The intuition for this result is the following: the incumbent suffers an electoral disadvantage whenever he does not satisfy voters' demands on the popular issue. He suffers the smallest disadvantage at the electoral campaign when he fully satisfied citizens' demands on the popular issue. However, this is a costly strategy for a policy motivated incumbent. If the incumbent is policy motivated he chooses the platform that forces him to compromise as little as possible whilst guaranteeing his reelection. But when the disalignment of interests with voters in the popular issue is too intense, this strategy becomes too costly and the incumbent prefers to ignore the citizens' proposal, implement his ideal policy and forgo reelection. As the electoral issue becomes more important, this scenario becomes even more likely, because electoral competition becomes more intense; the incumbent needs to satisfy voters more on the popular issue in order to avoid being disadvantaged, so that strategy becomes increasingly costly. At some point, he might prefer to implement his ideal policy on the popular issue and lose reelection.

Otherwise, the incumbent chooses in equilibrium a winning strategy that consists of a combination of policies that depends on the weight voters assign to his performance on each one of the issues. The larger the weight voters assign to the electoral issue, the more the incumbent concedes on that issue. Perhaps more surprisingly, this is not the case for the popular issue. The incumbent fully satisfies the voters' demands only when the weight citizens attach to the popular issue is neither too high nor too low. If citizens care a lot about the popular issue, this implies less competition in the electoral campaign. In this case, the incumbent does not need to fully satisfy the voters demands on the popular issue in order to be reelected. Similarly, when citizens do not care much about the popular issue, the incumbent can choose a policy close to his ideal one because his policy choice on the popular issue does not have a significant effect on his chances of reelection.

The remainder of paper is organized as follows. In the next section, we discuss the related literature. In Section 3, we discuss two real political mechanisms our analysis can apply to, namely referenda and participatory democracy. We apply our findings to the extension of experiences of participatory democracy in Brazil during the 90s and the early 00s. We also apply our results to the ambiguous effect of popular initiatives on the responsiveness of US legislators to citizens' policy preferences. Section 4 describes the formal model. Section 5 presents the results. The last section offers some concluding remarks. All technical proofs can be found in the Appendix.

2 Related literature

The present work is related to the analysis of the effect of popular initiatives on policy outcomes by Besley and Coate (2008). These authors study a citizen-candidate model with two policy issues, one of which can be subject to popular initiatives of the type that exist in many US states. Contrary to our analysis, these initiatives bind politicians if passed. These authors show that these type of policy proposals can sometimes improve the congruence between citizens' preferences and policy outcomes. As in our case, the final effect depends on the relative salience of the issues. However, Besley and Coate (2008) do not analyze how initiatives affect the reelection prospects of incumbent candidates.

Our model relates also to the literature on spatial competition with valence initiated by Stokes (1963) and later developed by Ansolabehere and Snyder (2000), Groseclose (2001) and Aragones and Palfrey (2002). In those models, one of the candidates holds an advantage due to exogenous non policy factors, called valence factors, such as charisma, better campaign funds or higher intelligence. The difference between our model and these is that in ours the origin of the advantage (if any) is endogenous. In our case, a good performance of the incumbent in the popular issue provides him with an advantage that has an effect on electoral competition similar to the one that valence factors have in the aforementioned models.

Let us mention at this point, that in some of these model of political competition with valence advantage there is a problem of nonexistence of Nash equilibria in pure strategies. This happens when candidates are mainly office motivated and the location of the median voter's ideal point is unknown to the candidates competing. Our model departs from those assumptions, and thus we can guarantee existence of a Nash equilibrium in pure strategies. In fact, we show by construction that there exist a Nash equilibrium in pure strategies, characterize its strategies and analyze its comparative statics. The reason is that on the one hand we have a game of complete information, and on the other hand we assume that candidates care not only about winning but also about the policy they implement or promise during the campaign. Thus we are able to show that in our case existence of Nash equilibrium in pure strategies is guaranteed.

In our model, the policy choice of the incumbent on the popular issue may also become a source of electoral disadvantage. This happens when the incumbent deviates too much from citizens' policy proposal on the popular issue. Hence, the incumbent faces a trade-off between ensuring reelection and respecting his own policy preferences. This trade-off that incumbents face in our model is similar to the one that emerges in dynamic settings with asymmetric information as in Reed (1994). In this context, Duggan (2000) and Banks and Duggan (2008) endogenize the trade-off between policy choices and re-election probabilities when elections are repeated, voters are fully rational, the challenger's preferences are privately known and policy spaces may be multidimensional.

Finally, there is a recent literature that relates incumbent accountability with pandering, that includes Canes-Wrone, Herron and Shotts (2001), Maskin and Tirole (2004) and Kartik, Squintani and Tinn (2012) among others. This literature focuses on the strategic choice of the incumbent who faces a problem of aggregation of information. Instead our model focuses on the strategic behavior of an incumbent that faces a problem of conflict of preferences. Thus, while complementing this literature, our results are more aligned with van Weelden (2013) because we also explore a complete information model and obtain non convergent policy outcomes in equilibrium.

3 Two sources of incumbency (dis)advantage

During their term in office, incumbents must make choices on new or common value issues. These choices might have a large negative effect on their chances of reelection if they are unpopular and citizens factor these choices into their evaluation of the incumbent's performance. Jeopardizing reelection may not be optimal even for purely policy motivated incumbents. The policy implemented in case they lose reelection may be worse for them than the policy which could have granted them victory. Therefore, incumbents may be willing to compromise on some dimension in order to be reelected

Two mechanisms that can generate this trade-off between policies and reelection chances are referenda and participatory democracy. The characteristics that both have in common are: (1) there is an issue that a significant part of the population considers to be very important; (2) the incumbent receives from citizens a policy proposal on this issue; (3) the incumbent must make a decision regarding that issue; (4) there is a significant proportion of voters who may base their vote on that issue. Next, we elaborate on how these two mechanisms fit in our main argument.

3.1 Referenda and popular initiatives

Facultative (non-mandatory) referenda may be initiated by a public authority or by some organized group of citizens. The latter case is known as popular initiative. Referenda may be either binding or non-binding. A non-binding referendum is merely advisory. It is left to the government or legislature to interpret its results and react to its outcome (even by ignoring it altogether). If the incumbent chooses not to implement the policy corresponding to the referendum outcome he may be punished by voters. Therefore, incumbents tend to follow the proposal emerged from the referendum.

The empirical evidence on the effect of referenda on congruence between policy outcomes and citizens' preferences is quite strong. Cross-section studies for Switzerland reveal that policy choices regarding provision of public goods correspond better with the preferences of voters in those cantons where referenda are more extensively used (Frey and Bohnet, 1993; Frey, 1994). Lutz and Hug (2006) run a cross-country study and find that the policy effects of referenda carry over to the national level. Our argument here is that referenda offer incumbents incentives to satisfy citizens' preferences because incumbents can obtain an advantage through them. However, we also show in our model that incumbents are not reelected if there exists a substantial disagreement between them and voters. Empirical evidence suggests that this disalignment between citizens and incumbents is frequent, especially in the case of local public services, as shown by Agreen, Dahlberg and Mork (2006) for a sample of Swedish municipalities. For the case of Switzerland, Frey and Bohnet (1993) report that 39% of the referenda held in that country between 1948 and 1990 yielded results that opposed the views of the Parliament.[3]

Still, a referendum initiated by the incumbent might have a weaker effect on voters' reaction than a referendum that originates with a popular initiative.[4] Popular initiatives sometimes take the form of legislative propos-

[3] More prominent examples are the two referenda called to decide whether the country should join the UN and the EU in 1986 and 1992 respectively, which yielded a majoritarian rejection (76% and 50%) despite the strong backing of all major political parties.

[4] Referenda called by the incumbent require them to perform strategy considerations

als that citizens can place in the ballot. This is the case in 24 US states, where petitions by citizens are voted after obtaining a number of signatures (between 2% and 15% of the voting population). Around 70% of the US population lives in either a state or a city in which initiatives are permitted. Popular initiatives are often regarded as an instrument that ensures a better congruence between citizens' preferences and policy outcomes (the so-called "gun behind the door effect"). Gerber (1996) and Matsusaka (2005, 2010) find indeed that laws passed in US states that allow citizen initiatives reflect more closely the preferences of their electorate, especially on specific issues such as abortion or gay rights (Arcenaux, 2002; Burden, 2005). However, other papers find that the alignment between citizens' ideology and adopted policies is identical in US states with and without initiatives (Lascher, Hagen and Rochlin, 1996; Camobreco, 1998). Our results can shed light on this mixed evidence. We find that when there is a substantial ideological disalignment between citizens and the politician in the issue the initiative is about, the politician may compromise on that issue just enough to ensure his reelection. In some occasions even, he may forgo reelection altogether. This would explain why the responsiveness of state politicians to citizens' initiatives does not seem to be strong.

Although popular initiatives are binding, the reelection chances of an incumbent candidate may still depend on whether the candidate endorses or not such proposals, and whether the incumbent implements and enforces the resulting policy (Gerber, Lupia and McCubbins, 2004). Some popular initiatives may even express discontent with the incumbent legislator or aim to weaken him. The available evidence suggests that their net effect is positive, and that popular initiatives do provide incumbents with an electoral advantage. Bali and Davis (2007) show that in those US states which permit popular initiatives, incumbent legislators enjoy a 1% to 2% higher chance of being reelected. These effects are small but significant and suggest that although citizens' proposals may constraint incumbents' discretion, they can be used by them to obtain additional electoral support.

3.2 Participatory democracy

Participatory democracy is an extended version of the system of representative democracy in which citizens make policy proposals through popular assemblies. Real cases of participatory democracy can be found in the town meetings of New England and in the village governance system of the Indian states of Kerala and West Bengal. But probably, the most well-known

as to when is optimal to called them. Xefteris (2011) analyzes this issue.

example is the system of municipal participatory budgeting system where popular assemblies coexist with formal political parties and local elections. Such system was implemented in all Peruvian municipalities after the constitutional reform of 2003, and has been adopted by nearly two hundred Brazilian municipalities. Among these, the pioneering experience is the one of Porto Alegre which started in 1989.[5] Participatory budgeting has also been applied to school, university, and public housing budgets.

In all these cases, popular assemblies and deliberation emerge as governance mechanisms because citizens are interested on a certain issue, normally a local one, and they would like certain policies to be implemented. Because they care enough about these issues, their expected benefits from participating in the process overcome the costs of coordinating in order to elaborate a policy proposal. Typically, a policy proposal emerges from these meetings and is submitted to the incumbent. But it is only advisory. The incumbent has formally complete discretion over the projects to prioritize and the policies to be implemented. However, because the support to these policy proposals is significant within the population, the incumbent's chances of being re-elected critically depend on his policy choices on that issue. For instance, Jaramillo and Wright (2015) report that mayors of Peruvian municipalities who chose to ignore the input of participatory fora typically suffered subsequent electoral defeats. Hence, the reaction of incumbent politicians to the outcome of participatory processes can award them an electoral advantage or a disadvantage.

A few years after the participatory budgeting system was first implemented in Porto Alegre, critics of the system claimed that it was being used as a partisan instrument by the ruling party, the Workers' Party. As a matter of fact, the party had won all municipal elections since 1989 by wide margins. Most studies indicate that the incumbent party did enjoy an advantage, as suggested by the higher levels of income redistribution and the patterns of citizen participation in the process (Aragones and Sanchez-Pages, 2009).

4 The model

We assume that electoral competition takes place across two dimensions, denoted by x and y. Each dimension is represented by the unit interval of the real line $[0, 1]$. Dimension x represents the popular issue and dimension y represents the electoral issue. There are two candidates: the incumbent and the challenger. The model proceeds in three stages. The first stage takes

[5] The implications of participatory democracy on the behavior of citizens and politicians and on policy outcomes are analyzed in Aragones and Sánchez-Pagés (2009).

place during the legislature: the incumbent receives a policy proposal on the popular issue and has to implement a policy on that issue. Both the policy proposed to the incumbent and the policy implemented by him on the popular issue are common knowledge to all candidates and all voters. The second stage is the electoral campaign: both candidates make policy announcements simultaneously on the electoral issue. Again all policy announcements are common knowledge to all candidates and all voters. It is assumed that the winner implements the announced policy on that issue. In the third stage of the game, the election takes place: voters decide whether to reelect the incumbent or vote for the challenger. The winner is selected by majority rule and implements the policy announced on the electoral issue.

4.1 Candidates

The two candidates are denoted by I for the incumbent and by C for the challenger. Candidates have single peaked preferences over the electoral issue y. Without any loss of generality, we assume that the ideal point of candidate I on the electoral issue is represented by $y_I = 0$ and the ideal point of candidate C is represented by $y_C = 1$. We assume that the incumbent has single-peaked preferences over the popular issue that are independent of his preferences on the electoral issue. The incumbent's ideal point on the popular issue is represented by $x_I = 0$. As we will argue below, it is not necessary to specify the preferences of the challenger over the popular issue.

Let us denote by $x(I)$ the policy chosen by the incumbent on the popular issue during the legislature. We assume the incumbent to be a unique decision maker. Thus, the present model applies to scenarios where the incumbent holds executive office, or to legislatures where a party holds a parliamentary majority and whose parliamentary representatives vote as a unified bloc.[6] Elections take place at the end of the legislature. When the electoral campaign starts, this choice $x(I)$ has already been made and it is taken as given. We model elections by means of a standard model of electoral competition on the issue y: the incumbent and the challenger simultaneously announce policy platforms denoted by $y(I)$ and $y(C)$ respectively. We assume full commitment, that is, the winner of the election implements on the electoral issue the policy he announced during the campaign.

We assume that candidates have preferences over policies but that they are also office-motivated. Candidates' payoffs depend on the policy chosen by the incumbent on the popular issue and the policy announcements of both

[6] Otherwise, it can be thought of as reduced form model of a more complex (and realistic) governmental system.

candidates on the electoral issue according to these utility functions:

$$U_I = -\left|x_I - x\left(I\right)\right| + \pi_I \left(K - \left|y_I - y\left(I\right)\right|\right) - \left(1-\pi_I\right)\left|y_I - y\left(C\right)\right|,$$

$$U_C = \left(1-\pi_I\right)\left(K - \left|y_C - y\left(C\right)\right|\right) - \pi_I \left(\left|y_C - y\left(I\right)\right|\right),$$

where $\pi_I = \pi_I(x(I), y(I), y(C))$ represents the probability that candidate I wins the election, and $1 - \pi_I$ denotes the probability that candidate C wins the election. The probability with which the incumbent is reelected depends on how the game unravels, that is, it depends on the policy choices made during the legislature (stage 1) and the policy announcements made during the campaign (stage 2).

K is a non-negative number that represents the utility or ego rent of holding office. $K = 0$ implies that candidates do not obtain any extra utility from holding office, they only derive utility from the policy implemented. In this case we would have two purely policy motivated candidates. The larger the value of K, the more candidates value being in office. Thus for larger values of K candidates care more about winning. When the value of K becomes arbitrarily large, candidates become purely office motivated.

Note that the incumbent obtains a negative payoff whenever he implements a policy on the popular issue that does not coincide with his ideal point on that issue. Observe also that because we assume that the challenger has no power over policy implementation on the popular issue before or after the election, the policy choice of the incumbent on that issue $x(I)$ has an non reversible impact on his payoffs. We elaborate more on this later.

For simplicity, we have assumed that the incumbent cares equally about the two issues. Introducing a parameter in the incumbent's payoff function that represents the relative weight that each issue has on the incumbent overall payoffs would not change qualitatively our results.

4.2 Voters

Voters have single-peaked preferences over the electoral issue y. We assume that their ideal points are uniformly distributed over y, so the ideal point of the median voter on the electoral issue is $y_m = \frac{1}{2}$. Let the ideal point of society in the electoral issue be denoted by $x_m > 0$. The parameter x_m can be interpreted as the outcome of a referendum or of a process of participatory democracy that took place before the beginning of the game. Notice that since the ideal point of the incumbent on the popular issue is assumed to be $x_I = 0$, the value of x_m measures the magnitude of the conflict of interests between the incumbent and the citizens with respect to the popular issue.

Here we assume that this proposal x_m is exogenous. In the final section of the paper, we discuss the consequences of endogenizing it.

When facing the election, voters observe the policies announced by both candidates on the electoral issue, $y(I)$ and $y(C)$, the policy implemented by the incumbent on the popular issue, $x(I)$, and then cast their vote. Voters use all the information available in order to evaluate the two candidates. Since they have different kinds of information about the performance of each candidate, their decision rule must exhibit some sort of asymmetry.

We assume that voter i evaluates the incumbent according to the function

$$V_i(I) = -(1-\mu)|x_m - x(I)| - \mu |y_i - y(I)|,$$

where μ is a parameter that measures the relative weight that voters assign to the electoral issue with respect to the popular issue. Notice that when $\mu = 1$ voters evaluation of the incumbent is not affected by his policy choice on the popular issue, thus our setup coincides with a standard one dimension electoral model. In this case we have that in equilibrium the incumbent will implement his ideal point on the popular issue, and both candidates will converge on the ideal point of the median voter on the electoral issue. On the other hand, when $\mu = 0$ voters evaluation of the incumbent only depends on his policy choice on the popular issue, thus his position on the electoral issue is complete irrelevant from the voters' point of view. In this case we have that in equilibrium the incumbent always prefer to win the election and he can do so by implementing a policy on the popular issue that satisfies the voters' preferences on that issue in a way that leaves the challenger no chance to affect the result.

In what follows we will focus on the setups corresponding to $0 \leq \mu \leq 1$. Notice that in all these cases voters combine the evaluation of the past performance of the incumbent on the popular issue with the evaluation of the expected performance of the incumbent on the electoral issue, that is, the voter's decision is determined by a combination of retrospective and prospective voting rules. Values of μ close to one mean that voters consider the electoral issue to be very important, and thus the prospective rule becomes more relevant on the voter's decision. In this case, voters' evaluation of the incumbent would not be much affected by his policy choice on the popular issue. Values of μ close to zero mean that the popular issue is regarded as very important by voters and thus the retrospective rule becomes more relevant on the voter's decisions. In this case, the voter's evaluation of the incumbent is strongly affected by his policy choice on that issue.

Note that voters evaluate the incumbent on the electoral issue by comparing his electoral platform, $y(I)$, to their own ideal point y_i. However, they evaluate the incumbent on the popular issue by comparing the policy he implemented, $x(I)$, to the policy proposed initially by citizens x_m. Hence, citizens consider the popular issue to be a common value issue and thus they measure the performance of the incumbent on the popular issue in an homogeneous way. This assumption is justified whenever the policy proposal x_m, represents the outcome of referenda, citizens assemblies or opinion polls, that is, when it represents the ideal policy on the popular issue of a substantial subset of the electorate. Our assumption of common evaluation of the incumbent's performance in the popular issue can alternatively be interpreted as an implicit commitment of citizens to punish politicians who do not follow the proposals submitted to them.

On the other hand, voter i evaluates the challenger according to the following function:

$$V_i(C) = -|y_i - y(C)|.$$

Voters evaluate the challenger according only to his promises on the electoral issue. The challenger could also make statements regarding the popular issue that might be incorporated by the voters in their evaluation. However, such statements are not actual facts as in the case of the incumbent, who had to implement a policy such as an annual budget, a reform of the abortion legislation, the participation or not in a war or the signature of an international treaty. We are then assuming that information about the popular issue, is only considered if it is hard information. We discuss the consequences of relaxing this assumption in the final section of the paper.[7]

Given voters' evaluations of both candidates, voter i will vote for candidate I if and only if

$$V_i(I) \geq V_i(C) \;\Leftrightarrow\; (1-\mu)|x_m - x(I)| + \mu|y_i - y(I)| \leq |y_i - y(C)|. \quad (1)$$

Notice that the lower the value of μ the more weight the incumbent's past choices have on the voter's incumbent evaluation, that is, the more retrospective their decisions become.[8] Low values of μ mean that voters barely base their decision on the past performance of the incumbent. As discussed in Section 2, the performance of the incumbent on the popular issue, i.e. the distance $|x_m - x(I)|$, has an effect on voters evaluations that is very similar to the effect of valence factors.

[7] Our evaluation functions imply that voters evaluate $y(C)$ and $y(I)$ differently. This could be solved by assuming $V_i(C) = -\mu|y_i - y(I)|$. Note however that in that case we would be imposing an incumbency disadvantage to start with.

[8] There exists a distinguished literature in which voters base their decisions on past performance of parties. Examples include Barro (1973), Ferejohn (1986), Austen-Smith and Banks (1989) and Reed (1994).

[Insert Figure 1 here]

Figure 1 shows how citizens' evaluation of candidates changes with the platforms announced by them at the electoral stage. If the two candidates were to propose the same platform the incumbent would suffer a disadvantage proportional to the distance between his choice $x(I)$ and the citizens' proposal x_m. On the other hand, citizens' evaluation of the incumbent is less sensitive to changes in his electoral platform $y(I)$. This feature implies that voters with ideal points at both extremes of the distribution may decide to vote for the same candidate. In fact, when the distance between the policy implemented $x(I)$ and the policy proposal x_m is large enough, the set of voters who decide to vote for the incumbent becomes non-connected.

Expression (1) implies that a citizen with ideal point $y_i = y(I)$ votes for candidate C whenever

$$\mu \leq 1 - \frac{|y_i - y(C)|}{|x_m - x(I)|}. \tag{2}$$

The set of voters who prefer to vote for the challenger but whose ideal policy y_i is closer to $y(I)$ enlarges as citizens care more about the popular issue and as the incumbent's choice departs more from their policy proposal x_m. This highlights that the existence of a policy proposal during the legislature can be a source of electoral disadvantage for the incumbent.

The present specification encompasses as particular cases some standard models of two-party competition. If $\mu = 1$, that is, if voters care only about the electoral issue, we have a standard model of electoral competition. In this case, for very large values of K candidates are purely opportunistic and the model describes a standard downsian framework. For relatively small values of K, candidates become mostly policy motivated, and our model reproduces Wittman's (1983) model of electoral competition. On the other hand, the case of $\mu = 0$, that is, the case where voters only care about the popular issue, boils down to a more general version of our previous work on participatory democracy (Aragonès and Sánchez-Pagés, 2009).

The incumbent is reelected if an only if the set of voters who prefer the incumbent to the challenger contains a majority of the population. We assume that if there is a tie, the incumbent is reelected. Since the decisions on the two dimensions of the model are made sequentially, one at each stage, we do not have to deal with the complexities of electoral equilibrium in a multidimensional space. In fact, we can solve it as a one dimensional model within each stage. In the next section we study the equilibrium of this game for all values of the parameters K, μ and x_m.

5 Equilibrium results

5.1 Electoral stage

In order to solve the model described above, we look for its subgame perfect equilibrium using backward induction. We start by analyzing the electoral stage, taking as given the choice of the incumbent on the popular issue. Citizens partially base their evaluation of the incumbent on his performance in the popular issue. He does not enter the election on the same grounds as the challenger. His choice on the popular issue has an impact on electoral competition, as the following lemma illustrates.

Lemma 1 *If* $y(I) = y(C)$, *then the incumbent obtains at least* $1-2|x(I) - x_m|$ *of the votes and the challenger obtains at most* $2|x(I) - x_m|$.

When both candidates choose the same position on the electoral issue, that is when $y(I) = y(C)$, only citizens at a distance of at least $|x(I) - x_m|$ from the policy proposed by both candidates vote for the incumbent. Thus, it is possible for the incumbent to capture the vote of extremists if he performs well enough in the popular issue, that is, when $|x(I) - x_m|$ is small enough. The incumbent's chances of being reelected are higher the less his policy choice in the popular issue $x(I)$ departs from the citizens' mandate x_m. As a matter of fact, there exists a threshold on this distance that is critical in determining whether the incumbent has an electoral advantage or not.

Lemma 2 *If* $|x(I) - x_m| \leq 1/4$, *then the incumbent wins in the equilibrium of the electoral stage. Otherwise, the challenger wins in equilibrium.*

The incumbent obtains a decisive advantage when he compromises enough on the popular issue. If, on the contrary, his policy choice on the popular issue departs considerably from the proposal x_m, then he has to compromise so much on the electoral issue in order to win that he rather prefers to lose.

Let us now fully describe the equilibrium at the electoral competition stage given a choice of $x(I)$. The following two lemmas characterize the strategies used by the winner of the election in equilibrium. These strategies define the equilibrium policy outcome of the electoral stage as well. First, we describe the equilibrium outcomes of the electoral stage for the case in which the incumbent is reelected in equilibrium. Notice that in this case for each parameter value under consideration the incumbent has a dominant strategy.

Lemma 3 *If* $|x(I) - x_m| \leq \frac{1}{4}$, *then the incumbent's dominant strategies at the electoral stage are:*

$$y^*(I) = \begin{cases} 0 & \text{if } |x(I) - x_m| \leq \frac{1-3\mu}{4(1-\mu)} \\ \frac{3\mu-1}{4\mu} + \frac{1-\mu}{\mu}|x(I) - x_m| & \text{otherwise.} \end{cases}$$

This proposition illustrates the trade-off that the incumbent faces. The more he pleases the electorate on the popular issue, i.e. the smaller $|x(I) - x_m|$, the closer his winning electoral platform can be to his ideal policy. The incumbent can even guarantee his reelection by implementing his ideal point on the electoral issue if he satisfies voters enough on the popular issue. In order to achieve this, he needs to compromise more on the popular issue the larger the weight voters put on the electoral issue μ (note that the bound $\frac{1-3\mu}{4(1-\mu)}$ decreases with μ).

Otherwise, if the incumbent implements a policy on the popular issue that departs significantly from the policy proposal x_m, then the incumbent still wins the election in equilibrium but his electoral platform $y^*(I)$ includes a certain degree of compromise. His electoral platform lies somewhere between his ideal point and the median voter's ideal point. It is closer to the median voter's ideal point the larger the distance between the policy he implemented in the popular issue $x(I)$ and the policy proposed on that issue x_m. This equilibrium policy choice is also closer to y_m the more weight voters put on the electoral issue, i.e. higher values of μ.[9] In the limit, when only the electoral issue is relevant, i.e. μ approaches to 1, the policy announced by the incumbent on the electoral issue coincides with the median voter's ideal point. By the same token, as the popular issue becomes more important, i.e. μ decreases, the policy announced by the incumbent on the electoral issue approaches the incumbent's ideal point.

The following lemma describes the equilibrium outcome of the electoral stage when the incumbent decides to forgo reelection. In that case, the equilibrium policy outcome in the electoral issue coincides with the strategies used by the challenger in the equilibrium of the electoral stage.

Lemma 4 *If $|x(I) - x_m| > \frac{1}{4}$, then the challenger's equilibrium strategy at the electoral stage is*

$$y^*(C) = \min\left\{\frac{1}{2} + \frac{1-\mu}{1+\mu}|x(I) - x_m|, 1\right\}.$$

When the incumbent has departed significantly from the citizens' ideal point in the popular issue, the challenger wins with a moderate policy in the resulting equilibrium of the electoral stage. Observe that $y^*(C)$ is decreasing in μ so, as before, the more important the electoral issue becomes the closer the policy outcome is to the median voter's ideal point. And the larger the distance between the policy proposal and the policy implemented on the popular issue, the closer the policy outcome on the electoral issue is to the challenger's ideal point.

[9] Straightfoward calculations show that $\frac{\partial y^*(I)}{\partial \mu} = \frac{1}{\mu^2}\left(\frac{1}{4} - |x(I) - x_m|\right) \geq 0$.

[Insert Figure 2 here]

Figure 2 shows the equilibrium policies for the two candidates, the voters' preferences over these policies and the position of the indifferent voter for two possible cases. In the above panel, the incumbent can implement his preferred policy in the electoral issue, i.e. $y^*(I) = 0$, because he implemented a policy very close to citizens' ideal policy in the popular issue. In the lower panel, the opposite holds; the incumbent implemented a policy far away from the citizens' ideal policy in the popular issue. This allows the challenger to win the election by proposing his most preferred platform in the electoral issue, i.e. $y^*(I) = 1$.

5.2 The popular issue

After solving for the equilibrium strategies of the electoral stage, we move backward in order to find the incumbent's optimal policy at the first stage.

Recall that when the incumbent is choosing which policy to implement on the popular issue he is facing a trade-off. If he implements a policy $x(I)$ that is relatively close to the citizens' proposal, x_m, he will be able to get reelected with an electoral platform relatively close to his ideal policy. The closer his choice $x(I)$ is to his own ideal policy on the popular issue, that is, the more his choice departs from x_m, the more he has to compromise on the electoral issue if he wants to remain in office. This strategy may be too costly if the incumbent is sufficiently policy motivated. Instead, he can implement his most preferred policy on the popular issue and forgo reelection. The next few results characterize this trade-off.

We define a best winning strategy for the incumbent to be a pair of policy choices for the popular and electoral issues such that they are optimal for the incumbent among all those strategies that guarantee that he wins the election. Similarly, we define a best losing strategy for the incumbent to be a pair of policy choices for the popular and electoral issues such that they are optimal for the incumbent among all those strategies that guarantee that he loses the election. First we find the best winning strategies and best losing strategies for the incumbent. Then we show under which conditions the incumbent prefers to be reelected.

The following Proposition describes the best winning policy choice of the incumbent given the importance of the electoral issue μ.

Proposition 1 *The incumbent's best winning strategies are:*

(i) When the electoral issue is of low importance, i.e. $\mu \leq \frac{1}{3}$*, to choose his*

ideal policy in the electoral issue, i.e. $y^*(I) = 0$, *and*

$$x^*(I) = \max\left\{x_m - \frac{1-3\mu}{4(1-\mu)}, 0\right\},$$

in the popular issue.

(*ii*) *When the electoral issue is of intermediate importance, i.e.* $\frac{1}{3} \leq \mu \leq \frac{1}{2}$, *to choose*

$$y^*(I) = \frac{3\mu - 1}{4\mu},$$

in the electoral issue and citizens' ideal policy in the popular issue, i.e. $x^*(I) = x_m$.

(*iii*) *When the electoral issue is more important than the popular issue, i.e.* $\mu \geq \frac{1}{2}$, *he chooses*

$$y^*(I) = \min\{\frac{1}{2}, \frac{3\mu - 1}{4\mu} + \frac{1-\mu}{\mu}x_m\},$$

in the electoral issue and

$$x^*(I) = \max\left\{x_m - \frac{1}{4}, 0\right\},$$

in the popular issue.

The relationship between the best winning electoral platform and μ is depicted in the lower panel of Figure 2. It is very intuitive: When the electoral issue is of low importance, the incumbent can set his preferred policy in that issue. But as the electoral issue becomes more important, i.e. μ goes up, the incumbent needs to select a platform closer to the median voter's ideal policy in order to win.

[Insert Figure 3 here]

Perhaps more surprising is the non-monotonic effect that the weight that citizens put on the popular issue has on the best winning policy that the incumbent can implement in that issue. It is depicted in the upper panel of Figure 2. When the popular issue is important (low μ) the incumbent is virtually facing no opposition. Citizens care virtually only about an issue in which the incumbent can act like a monopolist. The incumbent selects a policy in the popular issue that is closer to the citizens' ideal policy the more important the electoral issue becomes, i.e. the higher μ.

When μ becomes larger the incumbent has two options. He can either please citizens on the popular issue by implementing their proposed policy x_m, and in return choose a policy close to her ideal one on the electoral one. Or alternatively, he can pick the median voter's ideal policy on the electoral dimension and select a policy as close as possible to his own ideal one on the popular issue. For intermediate levels of μ, the first option is better because the electoral issue is still relatively unimportant so the incumbent can implement a policy relatively close to his ideal policy on that issue and still win. That winning electoral platform is less favorable for the incumbent the more important the electoral issue becomes, that is, the larger the value of μ. However, when the electoral issue is the most important one, i.e. $\mu > \frac{1}{2}$, the incumbent prefers the second option; he compromises substantially on the electoral issue, and implements the median voter's ideal point. In return, he departs from the citizens' ideal policy in the popular issue as much as he can without jeopardizing his reelection. In summary, the incumbent implements the citizens' policy proposal on the popular issue only when no issue is much more important than the other.

Next we characterize the incumbent's best losing strategy and the corresponding best response of the challenger.

Lemma 5 *The incumbent best losing strategies are* $x^*(I) = 0$ *and* $y^*(I) = \min\{\frac{1}{2} + \frac{1-\mu}{1+\mu}x_m, 1\}$ *which in turn implies that* $y^*(C) = y^*(I)$.

If the incumbent decides to forgo reelection, the best strategy that he can follow is to implement his preferred policy choice on the popular issue and to force the challenger to become as moderate as possible in the electoral one. In equilibrium, he announces the median voter's ideal policy and the challenger wins the election by announcing a platform that is closer to the median voter the more important the electoral issue is, i.e. the higher the value of μ.

The last step of the analysis amounts to characterize when the incumbent prefers to win the election given the best winning strategies and the best losing strategies described above. His incentives to remain in office depend on the level of disalignment with the population, measured by x_m, the relative weight that voters assign to the electoral issue measured by μ, and the value that the incumbent attaches to office K.

Proposition 2 *In equilibrium the incumbent wins*

(i) When there is little disalignment, i.e. $x_m \leq \frac{1}{4}$, *for any* $K \geq 0$ *and any* $0 \leq \mu \leq 1$.

(ii) *When disalignment is moderate. i.e.* $x_m \in [\frac{1}{4}, \frac{1+\mu}{8\mu}]$, *and the electoral issue is more important than the popular one, i.e* $\mu \geq \frac{1}{2}$, *for any* $K \geq 0$

(iii) *When disalignment is severe, i.e.* $x_m \geq \frac{1+\mu}{8\mu}$, *, and the electoral issue is more important than the popular one, i.e* $\mu \geq \frac{1}{2}$, *for*

$$K \geq -\frac{1}{4} + \frac{2\mu}{1+\mu} x_m.$$

When the preferences of the incumbent on the popular issue are aligned with those of society, i.e. $x_m \leq \frac{1}{4}$, the incumbent prefers to win (and thus wins) for all values of K and all values of μ. If the preferences of the incumbent on the popular issue are not aligned with the policy proposal x_m, the incumbent may decide to forgo reelection. He does so only when he is sufficiently policy motivated, i.e. for low enough values of K, and when the electoral issue is more important than the popular one. The first effect is clear so let us comment on the second. When the incumbent and the population have conflicting preferences on the popular issue, it is the more costly for the incumbent to please voters on that issue. As the electoral issue becomes more important, the incumbent needs to make larger concessions also in the electoral issue if he wants to be reelected. In the overall, satisfying voters in any dimension becomes very costly for the incumbent, so unless he is sufficiently office-motivated he prefers to lose.

Incumbents that are highly policy motivated, i.e. have low values of K, are more likely to suffer a disadvantage from being in office. They may find too costly to make a policy choice that guarantees their reelection when their preferences are not aligned with those of society preferences. The cost of being reelected may also be too high when the degree of competition on the electoral issue is high, i.e. μ close to 1. In that case, the incumbent has to propose a very moderate policy on the electoral issue if he wants to beat the challenger. If he does value policy strongly enough, he prefers to forgo reelection. Outside these cases, citizens' proposals can be used by the incumbent to obtain a decisive advantage in political competition and become reelected.

5.3 External validity

Let us now come back to the discussion in Section 3 about the real cases our model applies to. We now reappraise these cases at the light of the results obtained above.

For the case of Brazil, the Workers' Party won local elections in Porto Alegre by a landslide during the 1990s, to the extent that critics of participatory budgeting system blamed it as the source of such strong incumbency advantage. In 1998, the Workers' Party even gained control of Rio Grande do Sul, the state whose capital is Porto Alegre. Olivio Dutra, the new governor, introduced participatory budgeting at the state level. However, and in contrast with the success in Porto Alegre, Dutra was not reelected in the 2002 election. Our model suggests that the key for this different pattern lies at the higher degree of political competition at the state level. At the city level, the Workers' Party held strong support in Porto Alegre, and it is likely that the popular issue was dominant in voters' minds when casting their vote. However, at the state level, the Workers' Party faced a much stronger opposition and other issues, not covered by the participatory budgeting system, could have been very important. Our results show that a strong electoral competition erodes the advantage that the incumbent could enjoy due to participatory processes. Dutra could have lost his advantage in municipalities controlled by his Party due to his performance on local (popular) issues. Goldfrank and Schneider (2006) computed the difference between promised investments and actual investments completed for each municipality under the Workers' Party rule. These authors found a strong negative effect of these dashed expectations on the share of municipal votes of the Workers' Party in the 2002 election.

Our results corroborate to some extent research indicating that referenda in Swiss cantons help to make politicians implement policies closer to the most preferred by citizens. have less discretion. Feld and Matsusaka (2003) show that government spending in cantons with mandatory referenda on new projects is lower than in those cantons without referenda. Because mandatory referenda only apply when projects surpass certain spending thresholds, and incumbent politicians can split projects into smaller bits as a legal loophole. Therefore the reduction in spending the observe implies that incumbent spend more than what the median voter wants and that forms of direct democracy can bind politicians.

As mentioned in Section 2.1, the available evidence on the link between the presence of popular initiatives with politicians responsiveness in US states is much more ambiguous. Initiatives seem to work in making politicians adopt citizens' preferred policies in specific issues such as abortion or gay rights (Burden, 2005), although legislators can choose the extent of their compliance to the approved policies (Gerber et al., 2004). Still, our results can shed some light on this. Our model suggests that the reason behind this mixed evidence might be the interplay between political competition and the tension that exists between citizen's and politicians preferred policies. When citizens and politicians disagree strongly on the policy to be implemented, in other words, when the politician is sufficiently policy motivated, adopted policies on the popular issue may be very different from those the citizens want. If the popular issue is relatively unimportant, because, for instance, the election is very competitive, this effect should be stronger. We suggest that future studies on the responsiveness of politicians to the presence of popular initiatives should factor the closeness of past or incoming elections in order to control for this factor. Ideally, it would also be necessary to control for the importance voters attach to the issue the initiative refers to.

6 Concluding remarks

The main contribution of this paper is to study how electoral competition unravels when the policy choice of the incumbent in a pre-election issue factors into citizens' evaluation of his performance. We assumed that the performance of the incumbent on that issue is assessed by the distance between the policy proposed by citizens and the policy that the incumbent finally implemented. We characterized conditions under which the incumbents can use this pre-electoral issue to their advantage. In these cases, the incumbent has to adjust his policy choices in order to accommodate the policy proposal he receives, and the final policy outcome is relatively close to the policies most preferred by society. But this is not always the case. When the citizen's mandate is too far from the incumbent's preferred policy, the incumbent may decide to forgo reelection. In this case, the final policy outcome is typically bad from voters' point of view.

We have assumed that voters use an asymmetric rule in their evaluation of candidates. We identified two types asymmetries: 1) only the incumbent is responsible for the policy implemented on the popular issue, and 2) there is a policy proposal made only on the popular issue. Thus, we have assumed that voters evaluate the incumbent according to his performance on the two issues and the challenger only according to the platform he announces in the electoral issue. We could relax the assumption and assume instead that both candidates are evaluated on the two issues. The voters' evaluation of the challenger with respect to the popular issue would just become an exogenous parameter given that the challenger cannot implement any policy during the legislature. This parameter would represent the performance of the challenger in the popular issue in the past or its stated position on that issue. In that case, the evaluation rule in (1) would become

$$V_i(I) \geq V_i(C) \Leftrightarrow (1-\mu)\left|x_m - x(I)\right| + \mu\left|y_i - y(I)\right| \leq (1-\mu)\left|x_m - X\right| + \mu\left|y_i - y(C)\right|,$$

where X is the past performance of the challenger on the popular issue or its stated position. In that case, a citizen with ideal policy $y_i = y(I)$ will vote for candidate C whenever

$$\mu \geq 1 - \frac{|y_i - y(C)|}{|x_m - x(I)| + |y_i - y(C)| - |x_m - X|},$$

which is a simple modification of expression (2). If $|y_i - y(C)| > |x_m - X|$ then the challenger has an additional advantage compared to the case discussed in previous sections (a larger range of μ make the voter vote for C). This would be the relevant case when X is a statement. In that case, since it is cheap-talk to state an intention during the legislature, the challenger must choose $X = x_m$ in order to increase his advantage. If instead $|y_i - y(C)| < |x_m - X|$ the challenger has an additional disadvantage compared to our main model. That would correspond to the case where the challenger implemented a policy in the popular issue which was far from citizens' ideal policy x_m. In that case, citizens would evaluate him more unfavorably in the current election. But in summary, it is easy to see that such modification would not alter our results qualitatively.

Our results would also hold true if the weights voters use when evaluating the two candidates differed. As long as the weight that voters assign to the electoral issue when they evaluate the incumbent is smaller than the one they use to evaluate the challenger, our results would remain the same. Otherwise, the incumbent would suffer from a greater disadvantage but qualitatively our results would still go through.

Our modelling of citizens' evaluation rule constitutes a novel feature of our approach because it combines elements of both retrospective voting and prospective voting. Voters use retrospective voting to evaluate the performance of the incumbent with respect to the popular issue. And voters use prospective voting to evaluate the campaign promises that candidates announce during the electoral campaign. In order to use all the information available to them at the time of voting, voters combine these two different kinds of evaluations. Votes make use of past information regardless of whether past performance provide or not voters with information about the future choices of the incumbent. Our model can be interpreted as if citizens are fundamentally unhappy with an incumbent that deviated from the policy proposal they made to him during his time in office. This is despite the fact that the behavior of the incumbent in response to the proposal might be seen as a "sunk" choice during the electoral campaign. From this point of view, and given that any electoral advantage comes from that choice made by the incumbent, the present model might be seen as a behavioral model of endogenous valence.[10] Alternatively, we can interpret our reduced form model as if the incumbent has a type which determines how likely he is to break an electoral promise, and his performance on the popular issue carries some information about his type. Then when evaluating the incumbent, a rational voter would take into account both the policy implemented on the popular issue and his policy platform on the electoral issue, in which case our model can be thought of being based on pure rational assumptions.

The model assumes that these citizen policy proposals are exogenous. It could be extended by making them endogenous. This issue was partially addressed in Aragones and Sanchez-Pages (2009) for the case when the proposal comes from popular assemblies and only the popular issue is relevant for voters, i.e. μ close to zero. Results above show that policy proposals that are not aligned with the policy preferences of the incumbent tend to be neglected. This is more likely to be the case as electoral competition becomes stronger, i.e. as μ grows. Therefore, if policy proposals were endogenous, demands on the popular issue that were aligned with the preferences of the incumbent would be satisfied more likely when the intensity of electoral competition were intermediate. Given this, it might be optimal for voters to submit policy demands that do not put too much pressure on the incumbent.

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7 Appendix

Proof of Lemma 1. If $y(I) = y(C)$ then $V_i(I) \geq V_i(C)$ if and only if $-(1-\mu)|x_m - x(I)| - \mu|y_i - y(I)| \geq -|y_i - y(C)|$ which can be written as $|x_m - x(I)| \leq |y_i - y(C)|$. This implies that in this case I obtains votes from all i with ideal policies on the electoral issue at a distance from $y(C) = y(I)$ of at least $|x(I) - x_m|$, and that C can obtain at most $2|x(I) - x_m|$ votes. Therefore I obtains at least $1 - 2|x(I) - x_m|$ votes. Notice that C obtains exactly $2|x(I) - x_m|$ whenever $|x(I) - x_m| \leq y(I) = y(C) \leq 1 - |x(I) - x_m|$. ■

Proof of Lemma 2. First suppose that $|x(I) - x_m| \leq \frac{1}{4}$ and that in equilibrium $y(C) = y(I)$. Then C cannot win because by Lemma 1 C at most obtains $2|x(I) - x_m| \leq \frac{1}{2}$ votes. Suppose instead that $y(C) \neq y(I)$ and C wins. Then we must have

$$U_I(x(I), y(I), y(C)) = -x(I) - y(C).$$

Suppose that I chooses instead $y'(I) = y(C)$. Then by Lemma 1, I obtains at least $1 - 2|x(I) - x_m| \geq 1/2$ votes. Thus I wins and his utility is

$$U_I(x(I), y(C), y(C)) = -x(I) + K - y(C) \geq -x(I) - y(C),$$

so I can profitably deviate, and in equilibrium C cannot win. Therefore, we have proved that if $|x(I) - x_m| \leq \frac{1}{4}$ then I wins in the equilibrium of the electoral stage.

Now suppose that $|x(I) - x_m| \in (\frac{1}{4}, \frac{1}{2})$ and let us consider three cases:

- If $y(I) \in \left[\frac{1}{2} - |x(I) - x_m|, \frac{1}{2} + |x(I) - x_m|\right]$ suppose that $y(C) = y(I)$, then $V_i(I) \geq V_i(C)$ if and only if $|x_m - x(I)| \leq |y_i - y(C)|$ and C can obtain at most $2|x(I) - x_m| \geq \frac{1}{2}$ votes. By lemma 1 if $|x(I) - x_m| \leq y(I) = y(C) \leq 1 - |x(I) - x_m|$ then C obtains exactly $2|x(I) - x_m| > \frac{1}{2}$ votes and wins. Otherwise, suppose that $|x(I) - x_m| > y(I) = y(C)$ then C obtains $|x(I) - x_m| + y(I)$ votes and $|x(I) - x_m| + y(I) > \frac{1}{2}$. Similarly, suppose that $y(I) = y(C) > 1 - |x(I) - x_m|$ then C obtains $|x(I) - x_m| + 1 - y(I)$ votes and $|x(I) - x_m| + 1 - y(I) > \frac{1}{2}$. C prefers to do so since by mimicking I he obtains an extra payoff of K and his deviation does not involve any change in the policy finally implemented.

- If $y(I) \leq \frac{1}{2} - |x(I) - x_m|$ then C can defeat I with any $y(C) \in (\frac{3-2\mu}{4}, \frac{3}{4})$. To show this, note that the set of supporters of C is the interval $[\frac{y(C)+\mu y(I)}{1+\mu} - \frac{1-\mu}{1+\mu}\left|x\left(I\right)-x_m\right|, 1]$ whenever $y\left(C\right) > (1-\mu)(1-\left|x\left(I\right)-x_m\right|) + \mu y\left(I\right)$. In addition, this number of voters constitutes a majority if and only if $y\left(C\right) < \frac{1+\mu}{2} + (1-\mu)\left|x\left(I\right)-x_m\right| - \mu y\left(I\right)$. This defines an interval of platforms that C can use to defeat I. Given the restrictions on $\left|x\left(I\right)-x_m\right|$ and the assumption on $y(I)$, this interval is at least as large as the interval $(\frac{3-2\mu}{4}, \frac{3}{4})$. Hence, any platform in this interval guarantees C a victory against $y(I)$. Note again that C prefers to win rather than to let I win because, in addition to obtaining K, the policy outcome is closer to his ideal point.

- If $y(I) \geq \frac{1}{2} + \left|x\left(I\right)-x_m\right|$ then the best policy for C that guarantees a victory is $y(C) = \mu y(I) + (1-\mu)(\frac{1}{2} + \left|x\left(I\right)-x_m\right|)$. We show this by following the same procedure as above. First, we define the set of C's supporters and then check when it constitutes a majority. Next we check whether C actually wants to use this winning strategy. For this to be the case it needs to hold that

$$\begin{aligned} K - 1 + \mu y(I) + (1-\mu)(\frac{1}{2} + \left|x\left(I\right)-x_m\right|) &> -1 + y(I) \\ &\Leftrightarrow \quad y(I) < \frac{K}{1-\mu} + \frac{1}{2} + \left|x\left(I\right)-x_m\right|. \end{aligned}$$

  Hence, I will not able to win with a $y(I)$ in $(\frac{1}{2} + \left|x\left(I\right)-x_m\right|, 1]$ if $K > (1-\mu)(\frac{1}{2} - \left|x\left(I\right)-x_m\right|)$. If $K < (1-\mu)(\frac{1}{2} - \left|x\left(I\right)-x_m\right|)$ we need to check whether I prefers to win the election with such rightist policy. The best case scenario for I if he wants to win is when $y(I) = \frac{K}{1-\mu} + \frac{1}{2} + \left|x\left(I\right)-x_m\right|$. In that case, his payoff is just $-x(I) - \frac{\mu}{1-\mu}K - \frac{1}{2} - \left|x\left(I\right)-x_m\right|$. The best case scenario for I if on the contrary he decides to lose is to set $y(I) = \frac{1}{2} + \left|x\left(I\right)-x_m\right|$ given that that forces C to choose the same policy. His payoff is just $-x(I) - \frac{1}{2} - \left|x\left(I\right)-x_m\right|$, so he actually prefers to lose.

Finally, suppose that $\left|x\left(I\right)-x_m\right| \geq 1/2$. Suppose I is winning in equilibrium. Consider $y'(C)$ such that $y'(C) = y(I)$. Then C obtains more than half of the total vote. Thus C can win the election using this strategy. It is optimal for C to do so since he obtains an extra payoff of K and his deviation does not involve any change in the finally policy implemented. The reason for C winning is that if $y(I) \leq 1/2$ then C obtains a vote share equal to $y(I) + \min\left\{1-y(I), \left|x\left(I\right)-x_m\right|\right\}$ which is a majority. Similarly if $y(I) \geq 1/2$ then C obtains $1 - y(I) + \min\left\{y(I), \left|x\left(I\right)-x_m\right|\right\}$ which is also a majority. Thus I cannot win in equilibrium with $\left|x\left(I\right)-x_m\right| \geq 1/2$. ■

Proof of Lemma 3. We are going to describe the equilibrium strategies at the electoral competition stage given a choice of $x(I)$ that guarantees the incumbent's reelection, that is, $|x(I) - x_m| \leq \frac{1}{4}$. From the previous proposition we know that in this case I wins in equilibrium.

First we will prove that in equilibrium we must have $y(I) \leq y(C)$.

Suppose that $y(I)$ and $y(C)$ is an equilibrium outcome such that I wins and $y(C) < y(I)$. Then we must have $U_I(x(I), y(I), y(C)) = -x(I) + K - y(I)$. Consider that I chooses instead $y'(I) = y(C)$. Then by Lemma 1 I obtains at least $1-2|x(I) - x_m| > 1/2$ votes and his utility is $U_I(x(I), y(C), y(C)) = -x(I)+K-y(C)$. Since we assumed that $y(C) < y(I)$ then $U_I(x(I), y(C), y(C)) = -x(I) + K - y(C) > -x(I) + K - y(I) = U_I(x(I), y(I), y(C))$ thus contradicting that $y(C) < y(I)$ can be part of an equilibrium strategy. We must have $y(I) \leq y(C)$.

Next we characterize the sets of voters who vote for candidate I given $x(I)$, $y(I)$ and $y(C)$. We have to take care separately of three sets of voters: 1) those voters whose ideal points are to the left of the incumbent's policy choice on the electoral issue, i.e. $y_i < y(I)$; 2) those voters whose ideal points are to the right of the challenger's position on the electoral issue, i.e. $y_i > y(C)$; and 3) those voters whose ideal points are between the incumbent and the challenger's positions, i.e. $y(I) < y_i < y(C)$.

- The set of voters with $y_i \leq y(I)$ who vote for I consists of those voters with ideal policy y_i such that

$$y_i < \frac{y(C) - \mu y(I)}{1-\mu} - |x(I) - x_m| \equiv \underline{y}.$$

- Similarly, the set of voters with $y_i \geq y(C)$ who vote for I is given by voters with ideal policy y_i such that

$$y_i > \frac{y(C) - \mu y(I)}{1-\mu} + |x(I) - x_m| \equiv \overline{y}$$

  Clearly $\overline{y} \geq \underline{y}$. Since we have shown that $y(I) \leq y(C)$ it must be that $\frac{y(C)-\mu y(I)}{1-\mu} \geq y(C)$ so $\overline{y} \geq y(C)$. Notice that if $\underline{y} < 0$ then $\overline{y} < 1$ for all $|x(I) - x_m| < \frac{1}{2}$.

- Finally, the set of voters with $y(I) < y_i < y(C)$ who vote for I is given by those voters with ideal policy y_i such that

$$y_i < \frac{y(C) + \mu y(I)}{1+\mu} - \frac{1-\mu}{1+\mu}|x(I) - x_m| \equiv \widetilde{y}$$

Now, let us compare these thresholds. Again, using the fact that $y(I) \leq y(C)$ it must be that $\frac{y(C)+\mu y(I)}{1+\mu} \leq y(C)$ so $\widetilde{y} \leq y(C) \leq \overline{y}$. On the other hand, the comparison between $\underline{y}$ and $\widetilde{y}$ is not clear-cut. It is the case that $\underline{y} < \widetilde{y} < y(I)$ if and only if

$$y(C) - y(I) < (1-\mu)\left|x(I) - x_m\right|.$$

Thus, from the characterization of the sets of voters who vote for the incumbent we realize that two cases can emerge:

Case 1: If $y(C) - y(I) \geq (1-\mu)\left|x(I) - x_m\right|$ then we have that the votes that I obtains are given by $\widetilde{y} + \max\{0, 1-\overline{y}\}$.

Case 2: If $y(C) - y(I) < (1-\mu)\left|x(I) - x_m\right|$ then we have that the votes that I obtains are given by $\max\{0, \underline{y}\} + \max\{0, 1-\overline{y}\}$.

We will use this information in order to evaluate the performance of the incumbent's policy choices on the electoral issue. We will first analyse when it is the case that the incumbent can obtain a sure win by choosing his ideal point, i.e. $y(I) = 0$. Then we explore the rest of cases.

Let us start by assuming that $y(I) = 0$. Then the number of votes that I receives are

$$\#I = \begin{cases} 1 - \left|x(I) - x_m\right| - \frac{y(C)}{1-\mu} & \text{if } y(C) < (1-\mu)\left|x(I) - x_m\right| \\ 1 - \frac{2\mu}{1-\mu^2} y(C) - \frac{2}{1+\mu}\left|x(I) - x_m\right| & \text{if } y(C) \in \left[(1-\mu)\left|x(I) - x_m\right|, (1-\mu)(1 - \left|x(I) - x_m\right|)\right] \\ \frac{y(C)}{1+\mu} - \frac{1-\mu}{1+\mu}\left|x(I) - x_m\right| & \text{if } y(C) > (1-\mu)(1 - \left|x(I) - x_m\right|), \end{cases}$$

that attains a minimum when $y(C) = (1-\mu)(1 - \left|x(I) - x_m\right|)$. The number of votes in that case is still greater than $\frac{1}{2}$ if and only if

$$\left|x(I) - x_m\right| \leq \frac{1-3\mu}{4(1-\mu)}.$$

In summary, if this condition holds, then $y(I) = 0$ is a winning strategy for I no matter what $y(C)$ is. In that case, I can win by implementing his most preferred policy in the electoral issue. If the condition does not hold then there exists a platform $y(C)$ that can defeat $y(I) = 0$ and we need to consider further cases.

Let us suppose then for the rest of the proof that $\left|x(I) - x_m\right| > \frac{1-3\mu}{4(1-\mu)}$. We will now search for the best strategy of the incumbent in two separate intervals. First we will show that if $y(I) \in \left[0, \frac{3\mu-1}{4\mu} + \frac{1-\mu}{\mu}\left|x(I) - x_m\right|\right)$ then the incumbent cannot win. Then we will show that for $y(I) \in \left[\frac{3\mu-1}{4\mu} + \frac{1-\mu}{\mu}\left|x(I) - x_m\right|, \frac{1}{2}\right)$ we have that $y(I) = \frac{3\mu-1}{4\mu} + \frac{1-\mu}{\mu}\left|x(I) - x_m\right|$ is a dominant strategy. Notice that here we are only solving the candidates' problem of the electoral stage.

In this case, the policy choice on the popular issue is already a sunk cost for the incumbent. In addition we have already proved that we must have $y(I) \leq y(C)$, therefore the best strategy for the incumbent on the electoral issue must guarantee him a victory at the electoral stage.

- Let us first show that any platform $y(I) \in (0, \frac{3\mu-1}{4\mu} + \frac{1-\mu}{\mu} |x(I) - x_m|)$ can be defeated by $y(C) = \frac{3-\mu}{4}$. Notice that among all the strategies that defeat $y(I) \in (0, \frac{3\mu-1}{4\mu} + \frac{1-\mu}{\mu} |x(I) - x_m|)$ the one that maximizes C's payoffs is $y(C) = \frac{3-\mu}{4}$ First, note that when $y(C) = \frac{3-\mu}{4}$ we are in Case 1 above since

  $$y(C) - y(I) > (1-\mu) |x(I) - x_m| \Leftrightarrow y(I) < \frac{3-\mu}{4} - (1-\mu) |x(I) - x_m|$$

  and since we assumed that $y(I) \in (0, \frac{3\mu-1}{4\mu} + \frac{1-\mu}{\mu} |x(I) - x_m|)$. Simple algebra shows that

  $$\frac{3\mu-1}{4\mu} + \frac{1-\mu}{\mu} |x(I) - x_m| < \frac{3-\mu}{4} - (1-\mu) |x(I) - x_m| .$$

  One can also show that our assumption on $y(I)$ implies that $\overline{y_i} > 1$. All this together implies that the number of votes obtained by I is just $\widetilde{y}$ which in turn is smaller than $\frac{1}{2}$ if and only if

  $$y(I) < \frac{3\mu-1}{4\mu} + \frac{1-\mu}{\mu} |x(I) - x_m| ,$$

  which also holds by assumption. Hence, any I in this interval can be defeated if C chooses platform $y(C) = \frac{3-\mu}{4}$.

- Now suppose that $y(I) \in \left[\frac{3\mu-1}{4\mu} + \frac{1-\mu}{\mu} |x(I) - x_m| , \frac{1}{2}\right]$. Let us now show that in this interval $y(I) = \frac{3\mu-1}{4\mu} + \frac{1-\mu}{\mu} |x(I) - x_m|$ is a dominant strategy. Notice that if we show that I can win the electoral stage with $y(I) = \frac{3\mu-1}{4\mu} + \frac{1-\mu}{\mu} |x(I) - x_m|$ against any strategy of C then we are done, since given that he wins, this is the best strategy choice in the considered interval in terms of the incumbent's policy preferences.

  Thus, suppose that $y(I) = \frac{3\mu-1}{4\mu} + \frac{1-\mu}{\mu} |x(I) - x_m|$ and consider the following two cases:

  - $y(C) > \frac{3\mu-1}{4\mu} - \frac{1-\mu^2}{\mu} |x(I) - x_m|$. We will show that in this case, C cannot win. Notice that this case corresponds to the case 1 defined above ($y(C) - y(I) \geq (1-\mu) |x(I) - x_m|$ implies that votes for the incumbent are $\widetilde{y}+\max\{0,1-\overline{y}\}$) and thus the number of citizens who vote for the challenger are given by $\min\{1,\overline{y}\}-\widetilde{y}$. We need to consider two additional subcases depending on the value of the extremes of this interval.

i If $\overline{y}>1$ then C gets $1-\widetilde{y}$ votes and wins the election if and only if

$$\widetilde{y}<\frac{1}{2}\ \rightarrow\ y(C)<\frac{3-\mu}{4}$$

Since $\overline{y}>1$ if and only if $y(C)>\frac{3-\mu}{4}$ then this subcase cannot arise.

ii If $\overline{y}<1$ then C gets $\overline{y}-\widetilde{y}$ votes. This number of votes is greater than $\frac{1}{2}$ if and only if $y(C)>\frac{3-\mu}{4}$. Since $\overline{y}<1$ if and only if $y(C)<\frac{3-\mu}{4}$ again this case is not possible.

– $y(C)<\frac{3\mu-1}{4\mu}-\frac{1-\mu^2}{\mu}\left|x(I)-x_m\right|$. We will show that in this case, C cannot win. Notice that this case corresponds to the case 2 defined above ($y(C)-y(I)<(1-\mu)\left|x(I)-x_m\right|$ implies that votes for the incumbent are $\widetilde{y}+\max\{0,1-\overline{y}\}$) and simple algebra shows that this implies $\overline{y_i}<1$.Thus the challenger collects votes in the interval $[\max\{0,\underline{y}\},\overline{y}]$. Again, we need to consider then two different subcases:

i If $\underline{y}<0$ the challenger gets $\overline{y}$ votes and wins if and only if $y(C)\geq\frac{1+\mu}{4}$. But this leads to a contradiction because

$$\frac{1+\mu}{4}>\frac{3\mu-1}{4\mu}-\frac{1-\mu^2}{\mu}\left|x(I)-x_m\right|\ \Leftrightarrow\ \frac{1-\mu}{4(1+\mu)}>-\left|x(I)-x_m\right|,$$

which can never be the case. It cannot hold simultaneously that $y(C)<\frac{3\mu-1}{4\mu}-\frac{1-\mu^2}{\mu}\left|x(I)-x_m\right|$ and $y(C)\geq\frac{1+\mu}{4}$.

ii If $\underline{y_i}>0$ then C gets $\overline{y_i}-\underline{y_i}=2\left|x(I)-x_m\right|$ votes. So here C cannot win either since by assumption $\left|x(I)-x_m\right|\leq\frac{1}{4}$.

Thus C cannot win the election for any $y(C)$ he may choose whenever I chooses $y(I)=\frac{3\mu-1}{4\mu}+\frac{1-\mu}{\mu}\left|x(I)-x_m\right|$. Therefore, we have shown that I has two dominant strategies, $y(I)=0$ when $\left|x(I)-x_m\right|\leq\frac{1-3\mu}{4(1-\mu)}$ and $y(I)=\frac{3\mu-1}{4\mu}+\frac{1-\mu}{\mu}\left|x(I)-x_m\right|$ otherwise.

■

Proof of Lemma 4. We first consider the case with $\left|x(I)-x_m\right|\geq 1/2$, and afterwards we take care of the case with $1/4<\left|x(I)-x_m\right|<1/2$.

Suppose that $|x(I) - x_m| \geq 1/2$. If $y(I) > y(C)$ in equilibrium, consider $y'(C)$ such that $y'(C) = y(I)$ and notice that: 1) in this case C obtains more than $|x(I) - x_m|$ votes, that is, more than half of the total; and 2) the equilibrium policy outcome is larger, therefore better off for C. Thus this is a profitable deviation for C and it implies that $y(I) > y(C)$ cannot hold in equilibrium.

From the proof of Lemma 3 we know that if $y(I) \leq y(C)$ then C can only obtain the votes of those $y_i > \widetilde{y}_i$ and $y_i < \overline{y}_i$. Thus in order to obtain a majority of votes it is required that $\widetilde{y}_i < \frac{1}{2}$. This implies that

$$\widetilde{y}_i = \frac{y(C) + \mu y(I)}{1+\mu} - \frac{1-\mu}{1+\mu}|x(I) - x_m| < \frac{1}{2}$$

Thus the set of winnings strategies for C is defined by

$$y(C) < \frac{1+\mu}{2} + (1-\mu)|x(I) - x_m| - \mu y(I)$$

and among them C prefers the largest one $y(C) = \frac{1+\mu}{2} + (1-\mu)|x(I) - x_m| - \mu y(I)$.

Notice that for any strategy that I chooses, C can defeat it with $y(C) = \frac{1+\mu}{2} + (1-\mu)|x(I) - x_m| - \mu y(I)$. In this case, the best that I can do is to choose the strategy $y(I)$ that induces C to win with the policy that is most favorable for I, that is, the smallest possible. And we have that the value of $y(I)$ that minimizes $\frac{1+\mu}{2} + (1-\mu)|x(I) - x_m| - \mu y(I)$ and satisfies $y(I) \leq y(C)$ is $y(I) = y(C)$. Thus in equilibrium we will have $y(I) = y(C) = \min\left\{\frac{1}{2} + \frac{1-\mu}{1+\mu}|x(I) - x_m|, 1\right\}$.

Now suppose that $\frac{1}{4} < |x(I) - x_m| < \frac{1}{2}$. Here we will consider two cases: first we assume that $y(I) \in \left[0, \frac{1}{2} + |x(I) - x_m|\right]$ and then we assume that $y(I) \in \left[\frac{1}{2} + |x(I) - x_m|, 1\right]$

If $y(I) \in \left[0, \frac{1}{2} + |x(I) - x_m|\right]$ then C's best response, as in the previous proposition, is defined by $\widetilde{y}_i < \frac{1}{2}$. This implies that $\widetilde{y}_i = \frac{y(C)+\mu y(I)}{1+\mu} - \frac{1-\mu}{1+\mu}|x(I) - x_m| < \frac{1}{2}$. Thus the set of winnings strategies for C is defined by $y(C) < \frac{1+\mu}{2} + (1-\mu)|x(I) - x_m| - \mu y(I)$ and among them C prefers $y(C) = \frac{1+\mu}{2} + (1-\mu)|x(I) - x_m| - \mu y(I)$

And the best response for I in this case is the largest possible value of $y(I)$. Since in equilibrium we need to have $y(I) \leq y(C)$ then $y(I) \leq \frac{1+\mu}{2} + (1-\mu)|x(I) - x_m| - \mu y(I)$ implies $y(I) \leq \frac{1}{2} + \frac{1-\mu}{1+\mu}|x(I) - x_m|$. Thus for $y(I) \in \left[0, \frac{1}{2} + |x(I) - x_m|\right]$, C's best response is $y(C) = \min\left\{\frac{1}{2} + \frac{1-\mu}{1+\mu}|x(I) - x_m|, 1\right\}$.

If $y(I) \in \left[\frac{1}{2} + |x(I) - x_m|, 1\right]$ we have that C's best response is $y(C) \in \left[\frac{1}{2} + |x(I) - x_m|, 1\right]$, and for $y(I) \in \left[0, \frac{1}{2} - |x(I) - x_m|\right)$ we have that C's best response is $y(C) = \min\left\{\frac{1}{2} + \frac{1-\mu}{1+\mu}\left|x(I) - x_m\right|, 1\right\}$. Notice that the outcome of the election when $y(I) \in \left[0, \frac{1}{2} - \left|x(I) - x_m\right|\right)$ is better for I than the outcome that obtains when $y(I) \in \left[\frac{1}{2} + \left|x(I) - x_m\right|, 1\right]$. This implies that I's optimal strategy will not be in $\left[\frac{1}{2} + \left|x(I) - x_m\right|, 1\right]$.

Therefore the equilibrium if $\frac{1}{4} < \left|x(I) - x_m\right| < \frac{1}{2}$ is given by $y(I) = y(C) = \min\left\{\frac{1}{2} + \frac{1-\mu}{1+\mu}\left|x(I) - x_m\right|, 1\right\}$. ■

Proof of Proposition 1. Since we have already solved the equilibrium of the electoral stage game with the previous lemmata, now we go back to the first stage of the game, when the incumbent decides on his policy choice on the popular issue, and given the possible continuations of the game in equilibrium computed before, we will find the incumbent's optimal choices of $x(I)$.

Let us start with the case when $\left|x(I) - x_m\right| \leq \frac{1-3\mu}{4(1-\mu)}$. Notice that it can emerge only if $\mu \leq \frac{1}{3}$. From lemma 3 we know that in the electoral stage the incumbent has a best response that guarantees him a sure victory and it is $y^{*}(I) = 0$. The incumbent's payoff in this case will be $U_I = -x(I) + K$. In that case the incumbent's payoff is decreasing with $x(I)$ so his most preferred value of $x(I)$ in this range corresponds to the smallest possible value of $x(I)$, that is, $x(I) = \max\left\{x_m - \frac{1-3\mu}{4(1-\mu)}, 0\right\}$. Thus, the incumbent's payoff in this case will be $U_I = \begin{cases} K & \text{if } x_m \leq \frac{1-3\mu}{4(1-\mu)} \\ -x_m + \frac{1-3\mu}{4(1-\mu)} + K & \text{if } x_m \geq \frac{1-3\mu}{4(1-\mu)} \end{cases}$

When $\frac{1-3\mu}{4(1-\mu)} \leq \left|x(I) - x_m\right| \leq \frac{1}{4}$, from Lemma 3 we know that in the electoral stage the incumbent has a best response that guarantees him a sure victory and it is $y^{*}(I) = \frac{3\mu-1}{4\mu} + \frac{1-\mu}{\mu}\left|x(I) - x_m\right|$ thus his payoff in this case can be written as

$$U_I = -x(I) + K - \frac{3\mu-1}{4\mu} - \frac{1-\mu}{\mu}\left|x(I) - x_m\right| = -x_m + K - \frac{3\mu-1}{4\mu} - \frac{1-2\mu}{\mu}\left|x(I) - x_m\right|, \tag{3}$$

which is decreasing in $\left|x(I) - x_m\right|$ if and only if $\mu < \frac{1}{2}$. Therefore, we have that in order to maximize his payoff, I will choose the value of $x(I)$ that produces the smallest possible value of $\left|x(I) - x_m\right|$ whenever $\mu < \frac{1}{2}$, that is, $x(I) = \min\left\{x_m + \frac{3\mu-1}{4(1-\mu)}, x_m\right\}$. Notice that $x_m + \frac{3\mu-1}{4(1-\mu)} \geq x_m$ whenever $\mu \geq \frac{1}{3}$. Thus for $\frac{1}{3} \leq \mu < \frac{1}{2}$ we have that $x(I) = x_m$ and the incumbent's payoffs in this case are given by $U_I = -x_m + K - \frac{3\mu-1}{4\mu}$. Notice that if $y(I) = \frac{3\mu-1}{4\mu} + \frac{1-\mu}{\mu}\left|x(I) - x_m\right|$ and $\left|x(I) - x_m\right| = 0$ then $y(I) = \frac{3\mu-1}{4\mu}$. And for $\mu \leq \frac{1}{3}$ the incumbent chooses $x(I) = x_m + \frac{3\mu-1}{4(1-\mu)}$ so $y^{*}(I) = 0$ and his payoff is again $U_I = -x_m + \frac{1-3\mu}{4(1-\mu)} + K$.

Similarly when $\mu > \frac{1}{2}$, I will choose the value of $x(I)$ that produces the largest possible value of $|x(I) - x_m|$, that is, $|x(I) - x_m| = \frac{1}{4}$ and therefore $x(I) = \max\left\{x_m - \frac{1}{4}, 0\right\}$. Since $y(I) = \frac{3\mu-1}{4\mu} + \frac{1-\mu}{\mu}|x(I) - x_m|$, if $x_m \geq \frac{1}{4}$ then $x(I) = x_m - \frac{1}{4}$ and $y(I) = \frac{1}{2}$. If on the other hand $x_m < \frac{1}{4}$, then $x(I) = 0$ and $y(I) = \frac{3\mu-1}{4\mu} + \frac{1-\mu}{\mu}x_m$. The incumbent's payoffs in this case are given by

$$U_I = \begin{cases} -\frac{1-\mu}{\mu}x_m + \frac{1-3\mu}{4\mu} + K & \text{if } x_m < \frac{1}{4} \\ -x_m + K - \frac{1}{4} & \text{if } x_m \geq \frac{1}{4} \end{cases}$$

Finally, when $\mu = \frac{1}{2}$, the incumbent's payoffs in this case are invariant with his policy choice on the popular issue. His payoffs in this case are given by $U_I = -x_m + K - \frac{1}{4}$.

Therefore, in equilibrium we must have that

For $\mu \leq \frac{1}{3}$: $x^*(I) = \max\left\{x_m - \frac{1-3\mu}{4(1-\mu)}, 0\right\}$ and $y^*(I) = 0$.

For $\frac{1}{3} < \mu < \frac{1}{2}$: $x^*(I) = x_m$ and $y^*(I) = \frac{3\mu-1}{4\mu}$.

For $\mu \geq \frac{1}{2}$: $x^*(I) = \max\left\{x_m - \frac{1}{4}, 0\right\}$ and $y^*(I) = \min\{\frac{1}{2}, \frac{3\mu-1}{4\mu} + \frac{1-\mu}{\mu}x_m\}$.

■

Proof of Lemma 5. We know from previous results that if the incumbent decides to lose by setting $|x(I) - x_m| > \frac{1}{4}$, the challenger will win the election and set $y(C) = \min\left\{\frac{1}{2} + \frac{1-\mu}{1+\mu}|x(I) - x_m|, 1\right\}$. In that case, the incumbent receives the payoff

$$U_I = \begin{cases} -x(I) - \frac{1}{2} - \frac{1-\mu}{1+\mu}|x(I) - x_m| & \text{if } |x(I) - x_m| \leq \frac{1-\mu}{2(1+\mu)} \\ -x(I) - 1 & \text{if } |x(I) - x_m| \geq \frac{1-\mu}{2(1+\mu)} \end{cases}$$

which is always decreasing in $x(I)$. Therefore $x^*(I) = 0$, so for this case to be relevant $x_m \geq \frac{1}{4}$. This in turn implies that the challenger's best response is $y^*(C) = \min\left\{\frac{1}{2} + \frac{1-\mu}{1+\mu}x_m, 1\right\}$. Note that $y^*(C) = \frac{1}{2} + \frac{1-\mu}{1+\mu}x_m$ if and only if $x_m \leq \frac{1+\mu}{2(1-\mu)}$. This implies that $y^*(C) = \frac{1}{2} + \frac{1-\mu}{1+\mu}x_m$ if and only if $x_m \in [\frac{1}{4}, \frac{1+\mu}{2(1-\mu)}]$. Note that $\frac{1+\mu}{2(1-\mu)} > 1$ whenever $\mu > \frac{1}{3}$ so in that case $y^*(C) = \frac{1}{2} + \frac{1-\mu}{1+\mu}x_m$. ■

Proof of Proposition 2. Previous results show that since $x_m < \frac{1}{4}$ implies that $|x(I) - x_m| < \frac{1}{4}$ then I prefers to win in this case.

If $x_m \geq \frac{1}{4}$, from the proof of Lemma 5 we know that if the incumbent decides to lose then he receives payoffs:

if $\mu \leq \frac{1}{3}$: $U_I = \begin{cases} -\frac{1}{2} - \frac{1-\mu}{1+\mu}x_m & \text{if } x_m \leq \frac{1+\mu}{2(1-\mu)} \\ -1 & \text{if } x_m \geq \frac{1+\mu}{2(1-\mu)} \end{cases}$

if $\mu > \frac{1}{3}$: $U_I = -\frac{1}{2} - \frac{1-\mu}{1+\mu}x_m$

If the incumbent decides to use his best winning strategy then from the proof of Proposition 1 we know that his payoffs are:

if $\mu \leq \frac{1}{3} : U_I = \begin{cases} K & \text{if } x_m \leq \frac{1-3\mu}{4(1-\mu)} \\ -x_m + \frac{1-3\mu}{4(1-\mu)} + K & \text{if } x_m \geq \frac{1-3\mu}{4(1-\mu)} \end{cases}$

if $\frac{1}{3} < \mu < \frac{1}{2} : U_I = -x_m + K - \frac{3\mu-1}{4\mu}$

if $\mu \geq \frac{1}{2} : U_I = \begin{cases} -\frac{1-\mu}{\mu}x_m + \frac{1-3\mu}{4(1-\mu)} + K & \text{if } x_m \leq \frac{1}{4} \\ -x_m + K - \frac{1}{4} & \text{if } x_m \geq \frac{1}{4} \end{cases}$

The most complicated comparison occurs in the region $\mu \leq \frac{1}{3}$. Note that in that case $\frac{1-3\mu}{4(1-\mu)} \leq \frac{1+\mu}{2(1-\mu)}$

If $\mu \leq \frac{1}{3}$ and $x_m \leq \frac{1-3\mu}{4(1-\mu)}$ the incumbent would prefer to use winning strategies if and only if $K \geq -\frac{1}{2} - \frac{1-\mu}{1+\mu}x_m$ which always holds.

If $\mu \leq \frac{1}{3}$ and $x_m \in (\frac{1-3\mu}{4(1-\mu)}, \frac{1+\mu}{2(1-\mu)})$, the incumbent would prefer to use winning strategies if and only if $-x_m + \frac{1-3\mu}{4(1-\mu)} + K \geq -\frac{1}{2} - \frac{1-\mu}{1+\mu}x_m$, that is, $K \geq \frac{5\mu-3}{4(1-\mu)} + \frac{2\mu}{1+\mu}x_m$. Notice that $\frac{5\mu-3}{4(1-\mu)} + \frac{2\mu}{1+\mu}x_m \leq 0$ if and only if $x_m \leq \frac{(3-5\mu)(1+\mu)}{8(1-\mu)}$. The right hand side is decreasing in μ. Since for $\mu \leq \frac{1}{3}$ this implies that $\frac{(3-5\mu)(1+\mu)}{8(1-\mu)} \geq 1$ so it is always the case that $x_m \leq \frac{(3-5\mu)(1+\mu)}{8(1-\mu)}$. Therefore it holds that $K \geq 0 \geq \frac{5\mu-3}{4(1-\mu)} + \frac{2\mu}{1+\mu}x_m$. Therefore the incumbent prefers to use his winning strategy for all values of K.

If $\mu \leq \frac{1}{3}$ and $x_m \geq \frac{1+\mu}{2(1-\mu)}$ the incumbent would prefer to use his winning strategy if and only if $-x_m + \frac{1-3\mu}{4(1-\mu)} + K \geq -1$, which holds for all values of K.

If $\frac{1}{3} < \mu < \frac{1}{2}$ the incumbent would prefer to use his winning strategy if and only if $-x_m + K + \frac{1-3\mu}{4(1-\mu)} \geq -\frac{1}{2} - \frac{1-\mu}{1+\mu}x_m$, that is, if and only if $K \geq -\frac{3(1-\mu)}{4\mu} + \frac{2\mu}{1+\mu}x_m$. Notice that $-\frac{3(1-\mu)}{4\mu} + \frac{2\mu}{1+\mu}x_m \leq 0$ if and only if $x_m \leq \frac{3(1-\mu^2)}{8\mu^2}$. The right hand side of this expression is decreasing in μ. Then for $\mu \in (\frac{1}{3}, \frac{1}{2})$ we have that $\frac{3(1-\mu^2)}{8\mu^2} > \frac{9}{8}$, so it is always the case that $x_m \leq \frac{3(1-\mu^2)}{8\mu^2}$. Therefore it holds that $K \geq 0 \geq -\frac{3(1-\mu)}{4\mu} + \frac{2\mu}{1+\mu}x_m$ and the incumbent prefers to use his winning strategy for all values of K.

If $\mu \geq \frac{1}{2}$ and $x_m \geq \frac{1}{4}$ the incumbent would prefer to use his winning strategy if and only if $-x_m + K - \frac{1}{4} \geq -\frac{1}{2} - \frac{1-\mu}{1+\mu}x_m$, that is, if and only if $K \geq -\frac{1}{4} + \frac{2\mu}{1+\mu}x_m$. Notice that $-\frac{1}{4} + \frac{2\mu}{1+\mu}x_m \leq 0$ if and only if $x_m \leq \frac{1+\mu}{8\mu}$. The right hand side of this expression is decreasing in μ. Since $\mu \geq \frac{1}{2}$ we have that $\frac{1+\mu}{8\mu} \in \left[\frac{1}{4}, \frac{3}{8}\right]$. Hence this restriction is binding an the incumbent prefers to use his winning strategy for all values of K if and only if $x_m \leq \frac{1+\mu}{8\mu}(\leq \frac{3}{8})$. If $x_m \geq \min\{\frac{1+\mu}{8\mu}, 1\}$ then the incumbent prefers to use his winning strategy for $K \geq -\frac{1}{4} + \frac{2\mu}{1+\mu}x_m$.

If $\mu \geq \frac{1}{2}$ and $x_m \leq \frac{1}{4}$ the incumbent would prefer to use his winning strategy if and only if $-\frac{1-\mu}{\mu}x_m + \frac{1-3\mu}{4\mu} + K \geq -\frac{1}{2} - \frac{1-\mu}{1+\mu}x_m$, which holds for all values of K.

Thus we have that in equilibrium I will use a winning strategy in the following cases:

For all $K \geq 0$ and all $\mu \in (0,1)$ if $x_m \leq \frac{1}{4}$

For all $K \geq 0$ if $x_m \in \left[\frac{1}{4}, \frac{1+\mu}{8\mu}\right]$ and $\mu \geq \frac{1}{2}$

For $K \geq -\frac{1}{4} + \frac{2\mu}{1+\mu}x_m$ if $x_m \geq \frac{1+\mu}{8\mu}$ and $\mu \geq \frac{1}{2}$. ■

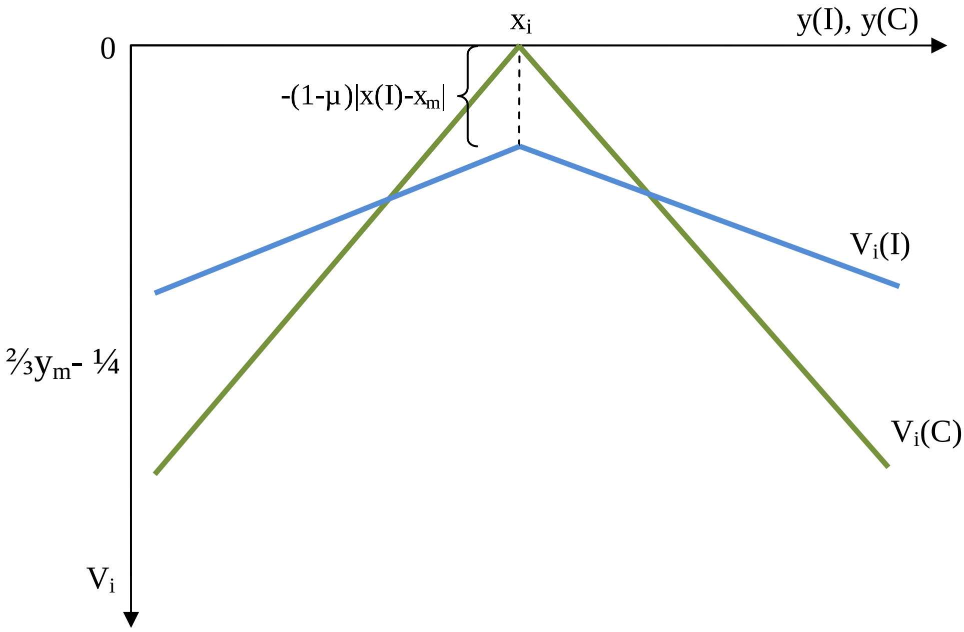

Figure 1: Voter's *i* evaluation of candidates at the electoral stage

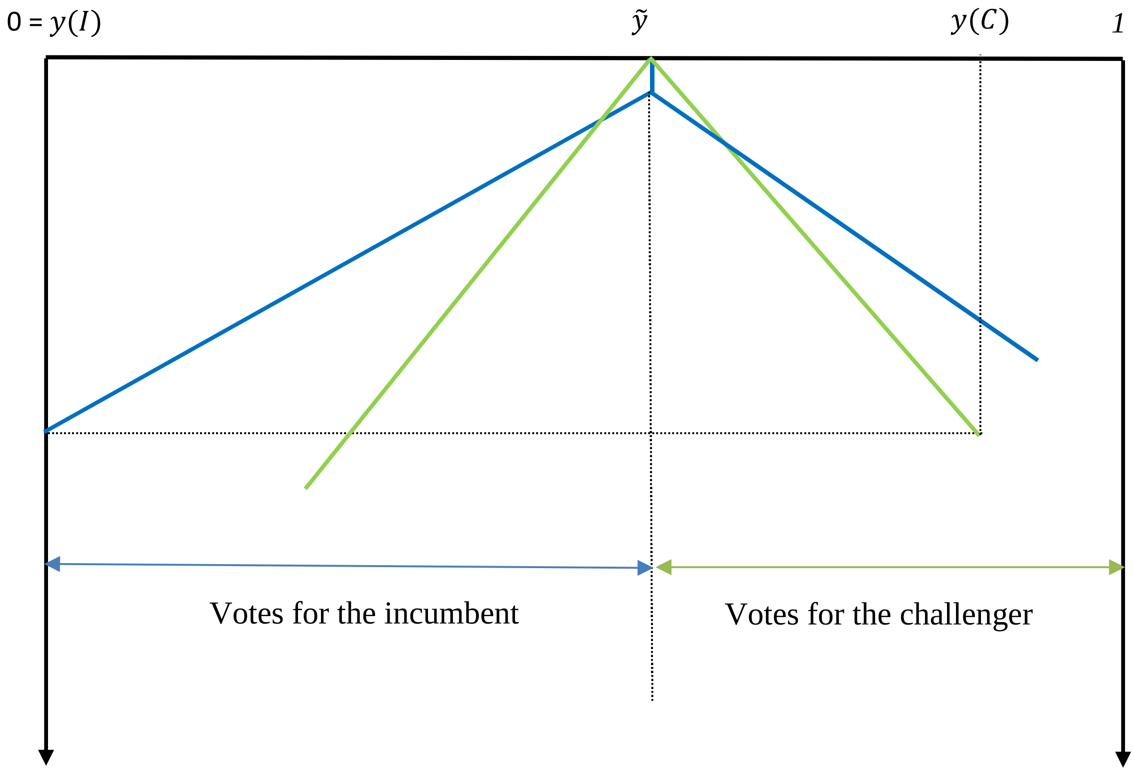

Fig 2a: When $|x(I) - x_m| < \frac{1-3\mu}{4(1-\mu)} (< \frac{1}{4})$ and $y(C) > (1-\mu)(x(I) - x_m)$

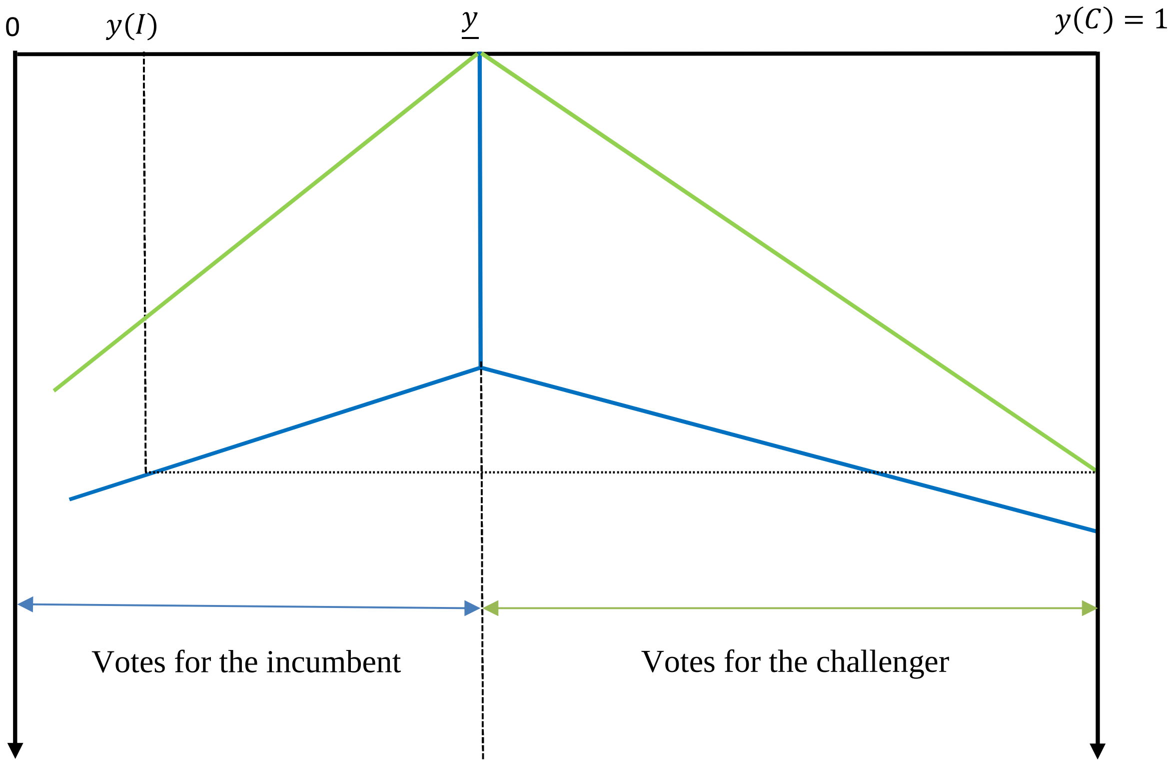

Fig 2b: When $|x(I) - x_m| > \frac{1+\mu}{2(1-\mu)} (> \frac{1}{4})$ and $y(I) > 1 - (1-\mu)(x(I) - x_m)$

Figure 2: Voters' preferences and electoral stage strategies

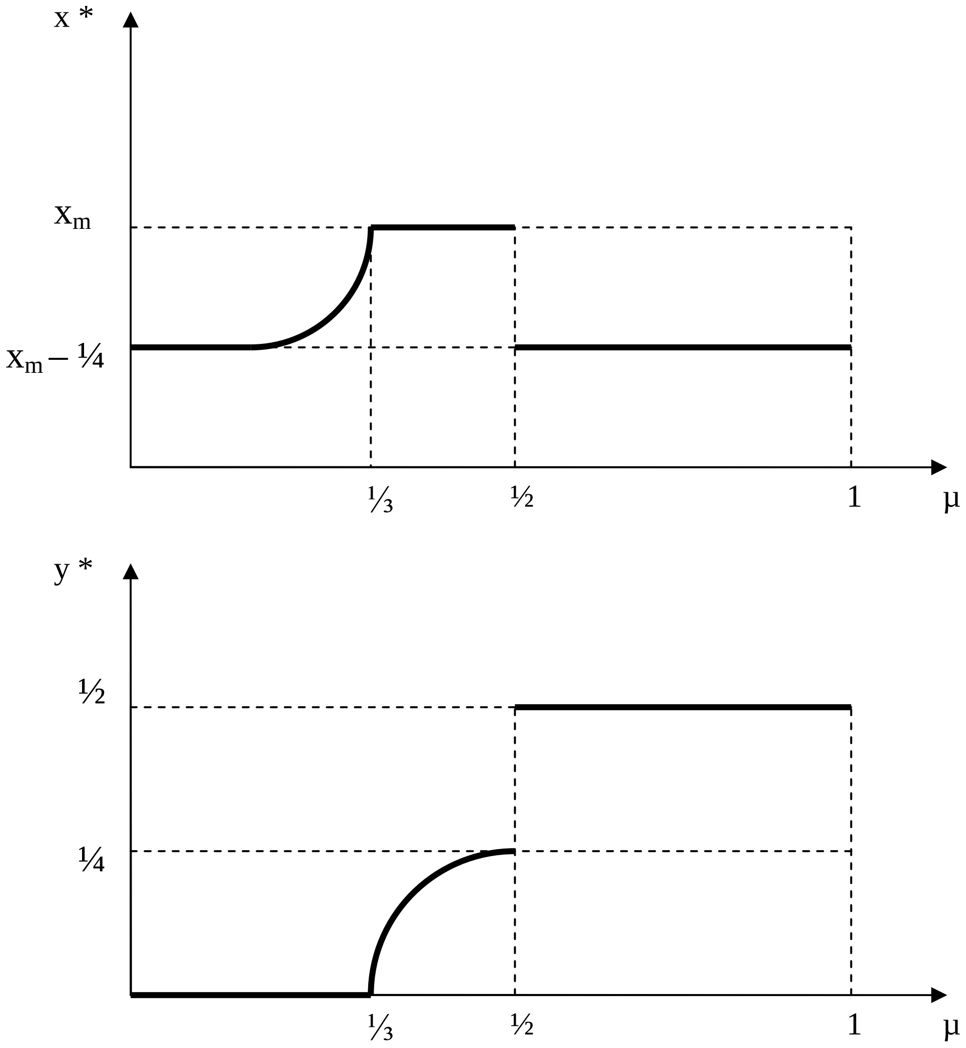

Figure 3: Incumbent's best winning strategies when $x_m > 1/4$.