3. Spatial Model of Elections

The median voter theorem is one of the central results in political science. It is the basic element of most models of rational choice on electoral competition.
This theorem claims that the policy that is most preferred by the median voter defeats by majority every other alternative in pair-wise comparisons, when alternatives are placed on one dimension. That is, the median voter's most preferred policy is the Condorcet Winner.
For more than one dimension, this result is only satisfied under very special conditions.
Spatial models of elections analyze the interplay of the decisions of candidates and voters. It is normally assumed that there are two candidates that maximize the probability of winning or the number of votes they obtain. Thus, we can represent electoral competition as a zero-sum game.

3.1 Spatial Model of Elections in one dimension

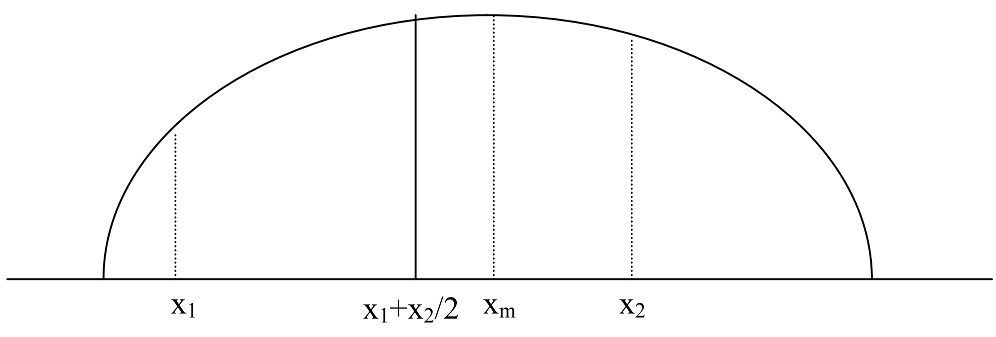
The policy space is assumed to be the real line or a real interval.

Classical references:
Downs 1957
Hotelling 1929

Each voter is assumed to have an ideal point in the policy space, x_i .
The utility of a voter is represented by the Euclidean distance between the ideal point and the proposed policy:

$$U_v(x) = -|x - x_i|$$

Voters decide to vote for the candidate whose proposed policy is closer to the voter's ideal point. If a voter is indifferent between two candidates, he votes for each of them with the same probability. It is assumed that all voters vote (no abstention).
Thus, the distribution of the voters' ideal points can be represented with a density function.



The area below the curve between two policies represents the proportion of voters whose ideal points are between those two policies.

For each two policies proposed by the two candidates, x_1 and x_2, the middle point $x_1+x_2/2$ represents the ideal point of the indifferent voter:

Suppose that the ideal point of the median voter is: x_m.
Then the utility that each candidate derives from a given pair of policies is:
(Notice that the candidates' only objective is to win the election.)

$$U_1(x_1,x_2)=\begin{cases} 1 & \text{if } |x_1-x_m|<|x_2-x_m| \\ 1/2 & \text{if } |x_1-x_m|=|x_2-x_m| \\ 0 & \text{if } |x_1-x_m|>|x_2-x_m| \end{cases}$$

$$U_2(x_1,x_2)=\begin{cases} 1 & \text{if } |x_1-x_m|>|x_2-x_m| \\ 1/2 & \text{if } |x_1-x_m|=|x_2-x_m| \\ 0 & \text{if } |x_1-x_m|<|x_2-x_m| \end{cases}$$

What are the candidates' equilibrium strategies?
(Notice that only candidates are the players of this game.)
Applying the median voter theorem: Both candidates choose the median voter's ideal point.

What if the distribution of the voters' ideal points is unknown to candidates, and they have identical beliefs that are common knowledge?

We can represent these beliefs with a probability function over the policy space, F(.). The probability assigned to each policy should be interpreted as the probability with which that policy coincides with the median voter's ideal point. Then, candidates maximize the probability of winning and the utility that each candidate derives from a given pair of policies if $x_1 < x_2$ is:

$$U_1(x_1,x_2)=F\left(\frac{x_1+x_2}{2}\right)$$

$$U_2(x_1,x_2)=1-F\left(\frac{x_1+x_2}{2}\right)$$

What are the candidates' equilibrium strategies?
Both candidates choose the expected median voter's ideal point: the median of the distribution F(.) and each candidate wins with probability 1/2.

A different interpretation of the same model: F represents the actual distribution of the voters' ideal points and candidates maximize the proportion of votes they obtain instead of the probability of winning.

What are the assumptions behind the median voter theorem?

Single-peaked preferences
All voters vote (no abstention)
Two candidates
One dimension
Candidates are office motivated
Candidates are identical

What if:

- Non single peaked preferences:
  as with more than one dimension
  → cycles
- abstention allowed:
  costly voting, probabilistic voting
  → new median voter.
- more than two candidates:
  exogenous number of candidates: Shepsle (1991)
  endogenous number of candidates: Shepsle (1991), Palfrey (1984)
  → 3.2
- more than one dimension:
  Plott (1967)
  McKelvey (1976)
  → 3.3
- candidates are policy motivated:
  with commitment: Wittman (1983)
  without commitment: Alesina (1988)
  → 3.4
- candidates are not identical:
  advantaged candidate: Aragones and Palfrey (2002)
  → 3.5 (maybe)

More than two candidates

Main results:
- Minimum Differentiation (convergence, median voter theorem)
- Maximal Differentiation (divergence)
- Duverger's Law

Two set-ups:
- Exogenous number of candidates: Shepsle (1991)
- Endogenous number of candidates: Greenber and Shepsle (1987), Palfrey (1984)

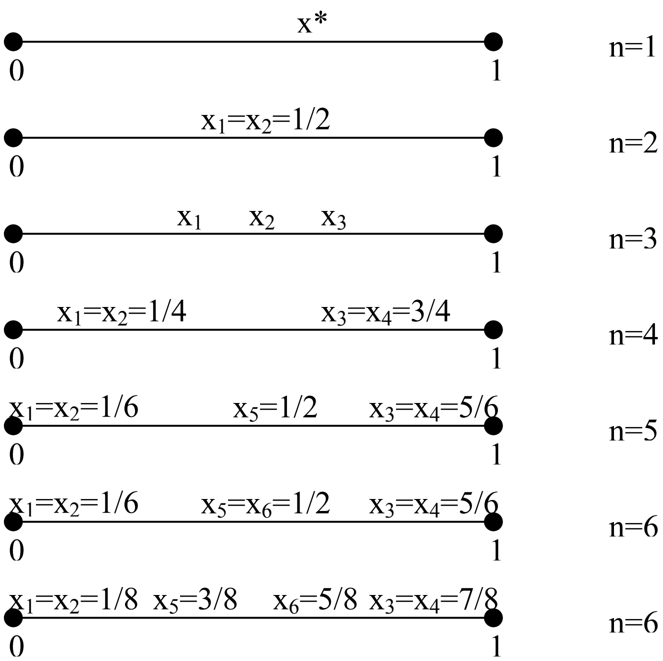
a) Exogenous number of candidates:

Assumptions:
Policy space = [0,1]
Candidates are vote maximizers
Winner determined by plurality rule

The necessary and sufficient conditions for equilibrium when the distribution of the voters' preferences is uniform are:

1) No electoral agent's support is smaller than any other agent's half-support.
2) Peripheral agents are paired.

For n = 1, 2, 4, 5, there is a unique equilibrium.
For n = 3 there is no equilibrium.
For n > 5 there is a continuum of equilibria.



If the distribution of the voters' preferences is not uniform we need additional conditions for existence:
For n = 1,2 there is an equilibrium.
For n = 3 there is no equilibrium
For n > 3 often there is no equilibrium.

b) Endogenous number of candidates: multi-seat contests.

Greenberg and Shepsle (1987)

This model focuses on multiseat contests in which K seats are at stake. A candidate is said to be elected if he or she finishes among the top K vote-getters. Thus the number of prizes is set exogenously, performance sufficient for reward is determined by competition endogenously, and success is measured by relative performance.

Assumptions:
Policy space = [0,1]
Voters have symmetric, single-peaked preferences and vote sincerely.
K seats = K winners
Winners determined by relative performance.
Candidates are not vote maximizers: want to be elected.
Two candidates cannot choose the same position.
Distribution of voters' preference is F(x)
Many entrants.
Zero costs to relocation.

With K seats at stake a candidate need only insure that it get fewer votes than no more than K-1 other candidates. Hence maximizing size of support may not be warranted. Greenberg and Shepsle suggest that rank maximization (or rank satisficing with K the level of aspiration) is the more suitable objective.

For a distribution of voters' preferences F(x), the measure of support of a candidate located at x is:
$s(x;A) = F[(a+a^+)/2]-F[(a+a^-)/2]$
Where A is the set of locations of the other candidates, a^+ and a^- are the locations of the candidate's right and left neighbors in A.

A K-equilibrium is a set A of K locations in [0,1] with the property that the support for a candidate locating in any one of these exceeds that garnered by a candidate locating at any other location in [0,1]. Thus, at any point in A a candidate wins (is among the top K vote-getters) because he cannot be displaced by a prospective entrant locating at some location not in A.

Formally a K-equilibrium is a set A such that:
1) A contains K candidates (locations) and
2) $s(b;A\cup b) \leq s(a;A\cup b)$ for all a in A and for all b not in A.

Thus, exiting candidates employ a Nash conjecture relative to existing rivals and a Stackelberg conjecture with regard to a prospective entrant. Thus, an existing candidate, in contemplating relocation, assumes his K-1 rivals will hold to their current locations, but that prospective entrant will optimize relative to the new configuration.
Impossibility theorem:
For every $K \geq 2$ there are societies satisfying the structure described above for which there is no K-equilibrium.

This result implies that the prospect of entry is potentially disequilibrating. The theorem does not assert that a K- equilibrium never exists, only that there are some configurations of voter preferences, for any K, for which equilibrium does not obtain.

Thus, if nevertheless one can observe stable electoral regularities these need to be explained by: barriers to entry, reputational effects, ….

Existence of K-equilibrium for some classes of distribution of voters' preferences:

1) (Cohen) Let f be a unimodal density function rescaled so that its mode is at zero. Suppose that f is continuous and symmetric with $f(0)<2f(w)$. Then $(-w,w)$ is a unique symmetric 2-equilibrium.
2) (Weber) Let $3 \leq K \leq n$.
   A K-equilibrium exists for all societies with n voters iff $n \leq 2K + 1$.
   For $K = 2$ a K- equilibrium exists for all societies of fewer than 7 voters.

Weber shows that K-equilibria can be guaranteed to exist so long as the electorate is small relative to the number of seats at stake. While of interest to the politics of committees, this result obviously has less bearing on mass electoral systems.
In effect Weber's result asserts that whenever the number of seats up for election is fewer than about half the electorate, a K-equilibrium cannot be guaranteed: existence will depend on voters' preferences.

c) Endogenous number of candidates: decision to enter.

Palfrey (1984)
Assumptions:
Policy space = [0,1]
Voters have symmetric, single-peaked preferences and vote sincerely.
Candidates are vote maximizers
Winner determined by plurality rule
Distribution of voters' preferences is F(x)
Two established candidates and one prospective entrant
Zero costs to relocation.

Suppose that each established candidate makes a Cournot-Nash conjecture about his established opponent and a Stackelberg conjecture about the entrant: the two established candidates choose simultaneously and later the prospective entrant makes his choice knowing it opponents policy choices.

A limit equilibrium is a location vector (x, y, z(x,y)) (where z is the entrants' location) such that:

u_x (x, y, z(x,y)) ≥ u_x (x', y, z(x',y))

u_y (x, y, z(x,y)) ≥ u_y (x, y', z(x,y'))

u_z (x, y, z(x,y)) ≥ u_z (x, y, z'(x,y))

There is a unique limit equilibrium:

If the distribution of the voters' preferences is uniform the equilibrium is: ¼, ¾, and the entrant in (1/4,3/4).

If the distribution of the voters' preferences is not uniform:

F(x*) = 1-2F(1/4+x*/2)

F(y*) = 1-F(x*)

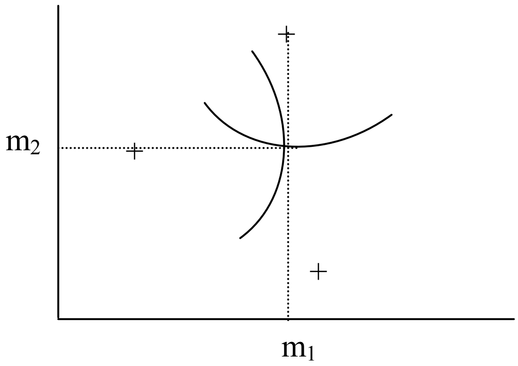
z* (the entrant) in a vote maximizing location between x* and y*

The entrant can never win in a limit equilibrium.

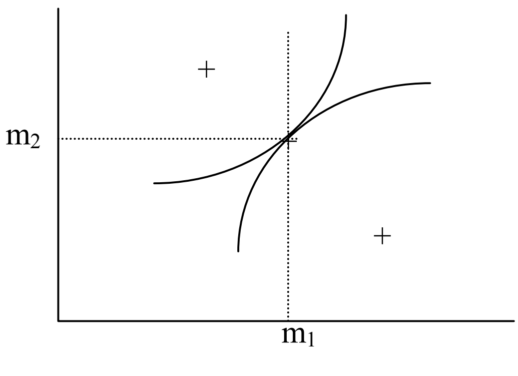
"It may be argued that the prospective entrant need not actually enter, but only threaten to enter to induce Palfrey's limit equilibrium. To make such a threat credible, however, it seems that either the entrant must have some chance of winning or her entry must have some chance of producing a winning location she prefers to the convergent equilibrium that would otherwise occur. If either of these conditions were satisfied, then the threat, possessing credibility, would never have to be implemented, i.e., prospective entry would have an effect without actually being observed."

More than one dimension

Suppose there are two dimensions, then the policy space is given by a plane.
If preferences are separable then indifference curves are circumferences,
Where is the median voter ? decisive voter?



Suppose that there are three voters.
We can find the median voter in each dimension. In general they differ and the selected policy is not an equilibrium: the policies between the two curves dominate it.
Two general results:
Normally, there is no equilibrium in pure strategies.
This equilibrium only exists under very special symmetry conditions.
Normally, there is an equilibrium in mixed strategies.



The main issue in a multidimensional policy space is the potential for chaos.
McKelvey's Chaos theorem (1976): if one actor could control the agenda of proposals voted on, he could construct a sequence of proposals that would take the outcome from any point in the policy space to almost any other point.
Degree of chaos: Pareto set.
Plott (1967): Core may be not empty for specific configurations.

Candidates' objectives (1)

If candidates are only policy motivated: each proposes its own ideal point, and the winner is the candidate with the ideal point closer to the median voter's ideal point.
If candidates are only office motivated : both propose the median voter's ideal point, and the outcome of the election is a tie.
If candidates care about winning the election and also the policy they implement (not the policy implemented by the opponent):

$$U_A(x_A) = \alpha \Pr(A \; wins : x_A, x_B) - (1-\alpha)|x_A - y_A|$$

where y_A represents candidate A ideal point, and α represents the relative weight that candidate A assigns to winning.
Suppose that the policy space is the interval [0,1] and the beliefs of candidates' over the location of the median voter's ideal point are represented by a uniform distribution over the policy space, which implies that the expected location of the median voter's ideal point is ½.
A parallel interpretation: candidates care about the mount of votes they obtain, not only about winning the election and there is complete information. The distribution of the voters' preferences is represented by a uniform distribution over the policy space which implies that the location of the median voter's ideal point is ½.
Consider two candidates such that y_A=0 and y_B=1. We can compute their optimal strategies:

$$\max_{x_A} U_A(x_A, x_B) = \alpha \frac{x_A + x_B}{2} - (1-\alpha)x_A$$

$$F.O.C.: \frac{\partial U_A(x_A, x_B)}{\partial x_A} = \frac{\alpha}{2} - (1-\alpha) > 0 \quad iff \quad \alpha > \frac{2}{3}$$

$$\max_{x_B} U_B(x_B, x_A) = \alpha\left(1 - \frac{x_A + x_B}{2}\right) - (1-\alpha)(1-x_B)$$

$$F.O.C.: \frac{\partial U_B(x_B, x_A)}{\partial x_B} = -\frac{\alpha}{2} + (1-\alpha) > 0 \quad iff \quad \alpha < \frac{2}{3}$$

Thus, their optimal replies are:

$$x_A(x_B) = \begin{cases} x_B & if \quad \alpha \geq \frac{2}{3} \\ 0 & if \quad \alpha \leq \frac{2}{3} \end{cases} \qquad and \qquad x_B(x_A) = \begin{cases} x_A & if \quad \alpha \geq \frac{2}{3} \\ 1 & if \quad \alpha \leq \frac{2}{3} \end{cases}$$

And in equilibrium we have:

$$x_A = x_B = x_{median} = \frac{1}{2} \quad if \quad \alpha \geq \frac{2}{3}$$

$$x_A = 0, x_B = 1 \quad if \quad \alpha \leq \frac{2}{3}$$

When winning is very important relative to policy, $\alpha \geq 2/3$, in equilibrium both candidates propose the median voter's ideal point. When policy is very important relative to winning, $\alpha \leq 2/3$, in equilibrium each candidate proposes his ideal point. If $\alpha = 2/3$ both equilibria coexist.

Candidates' objectives (2)

If candidates are only policy motivated: each proposes its own ideal point, and the winner is the candidate with the ideal point closer to the median voter's ideal point.
If candidates are only office motivated : both propose the median voter's ideal point, and the outcome of the election is a tie.
If candidates care about both, policy and office:

Suppose that there are two candidates, 1 and 2, that compete in elections repeatedly. Candidates' utility from policy is single-peaked and concave and it is given by:

$U_i(x) = \sum_{t=0}^{\infty} q^t u_i(x_t)$ where $u_1(x) = -\frac{(x-c)^2}{2}$ and $u_2(x) = -\frac{x^2}{2}$.

That is, x_t represents the policy implemented in period t, and q represents the discount factor, with $0 < q < 1$. The ideal point of candidate 1 is $c > 0$ and the ideal point of candidate 2 is 0. The winner of a given election obtains an additional payoff of $k > 0$.

Voters have preferences on x, they are assumed to be rational and forward looking. They know the candidates' objective. Candidates do not know what the median voter's ideal point is and their beliefs are identical and common knowledge and they are represented by $P(x_1^e, x_2^e)$, the probability that candidate 1 wins when voters expect that if candidate 1 wins he implements policy x_1^e, and voters expect that if candidate 2 wins he implements x_2^e.
We assume that $P(x_1^e, x_2^e)$ decreases with x_1^e iff $x_1^e > x_2^e$; and it decreases with x_2^e iff $x_1^e > x_2^e$.

Timing:
Both candidates announce simultaneously their policies: x_1^a and x_2^a.
Voters update beliefs on candidates expected policy implementation:
$x_1^e = x_1^a$ or $E(x_1:I_{t-1})$; and $x_2^e = x_2^a$ or $E(x_2:I_{t-1})$;
Voters vote.
The winner decides which policy to implement.

a) Wittman (1983): policy convergence.
One shot game with binding commitments: $x_i^a = x_i^e = x_i$ for all i.

$x_1 = \text{argmax } P(x_1, x_2)(u_1(x_1)+k)+(1-P(x_1, x_2))\, u_2(x_2)$
$x_2 = \text{argmax } P(x_1, x_2)\, u_2(x_1)+(1-P(x_1, x_2))(u_2(x_2)+k)$

If $P(x_1, x_2)$ is concave in x_1 and convex in x_2, then there exists a unique equilibrium. In this equilibrium: $0 < x_2 < x_1 < c$ and the distance between x_1 and x_2 decreases with k. Notice that k represents how much the candidates care about office relative to the policy implemented. When k becomes very large then we obtain full convergence: candidates are basically office seeking.

b) Alesina (1988): policy divergence.
One shot game without binding commitments: $x_i^a \neq x_i$ and then $x_i^a \neq x_i^e$ for all i.

$x_1 = \text{argmax } P(x_1, x_2)(u_1(x_1)+k)+(1-P(x_1, x_2))\, u_2(x_2)$
$x_2 = \text{argmax } P(x_1, x_2)\, u_2(x_1)+(1-P(x_1, x_2))(u_2(x_2)+k)$

For all k > 0 there exists a unique equilibrium. In this equilibrium: $x_2 = 0$, $x_1 = c$, $x_1^e = c$, $x_2^e = 0$ and P(c,0).

We have seen that if there are no pre-commitments, all we have is policy divergence. Thus, policy convergence relies on pre-commitments.

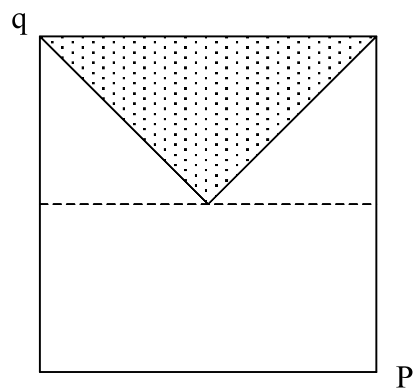
But, on the other hand if we assume that P(c,0) = P(c/2,c/2) = 1/2
$EU_1(c,0) = P(c,0)\, k + (1-P(c,0))\, (-c^2/2) = k/2 - c^2/4$
$EU_1(c/2,c/2) = P(c/2,c/2)\, (-c^2/8 + k) + (1-P(c/2,c/2))\, (-c^2/8) = k/2 - c^2/8$
We have that candidate1 prefers full convergence to full divergence because $EU_1(c,0) < EU_1(c/2,c/2)$, because of concavity.

In this case the efficient frontier is given by $x_1 = x_2 = \lambda c$ where λ represents the weight assigned to party 1.
The relevant part of the efficient frontier is the segment in which both parties are better off than in the one-shot Nash equilibrium (individual rationality constraint):
$1-\sqrt{1-P(c,0)} \leq \lambda \leq \sqrt{P(c,0)}$

c) Alesina (1988):
Infinitely repeated game allows to obtain credibility of announcements through reputation: make deviations costly.
Obtain partial convergence.
Level of convergence or polarization depends on concavity of objective function and discount factor.

Equilibrium:
Candidates announce convergent policies. Voters believe them. Announcement is implemented.
If a candidate deviates (implements his ideal point) from then on everybody (voters and opponent) believe that he will implement his ideal point: revert to one-shot Nash equilibrium.

Equilibrium condition to implement first best:
Incentive to deviate < cost of deviating.
- need q large enough (from folk theorems)
- easier if P(c,0) is around ½ (balanced system: otherwise one party has little bargaining power, and the selected policy is far from his ideal point, thus, he has a large incentive to deviate.)



Reminder. q = discount factor.
If q is low, the first best cannot be achieved:
Unique equilibrium for $q = 0$ is $x_1 = c$ and $x_2 = 0$ (policy divergence).

If $q > 0$ there are two equilibria:
- one-shot Nash equilibrium (policy divergence)
- $x_1(q,c)$ and $x_2(q,c)$ with $0 < x_2(q,c) < x_1(q,c) < c$ which Pareto improves upon the one-shot Nash equilibrium.

References

Enelow, James and Melvin Hinich. 1984. "*The spatial theory of voting: an introduction*" New York: Cambridge University Press.
Enelow, James and Melvin Hinich. 1984. "*Advances in the spatial theory of voting*" Cambridge: Cambridge University Press.

Additional references:
Alesina, Alberto (1988) "Credibility and Policy Convergence in a two-Party System with Rational Voters" *American Economic Review* 78(4): 796-805.
Aragones, Enriqueta and Thomas Palfrey (2002) "Mixed Equilibrium in a Downsian Model with Favored Candidate" *Journal of Economic Theory* 103:131-161.
Downs, Anthony (1957) *An Economic Theory of Democracy*, New York: Harper and Row.
Enelow, James and Melvin Hinich. 1984. "*The spatial theory of voting: an introduction*" New York: Cambridge University Press.
Enelow, James and Melvin Hinich. 1984. "*Advances in the spatial theory of voting*" Cambridge: Cambridge University Press.
Greenberg, J. and Kenneth Shepsle (1987) "The Effects of Electoral rewards in Multiparty Competition with Entry" *American Political Science Review* 81: 525-537.
Hotelling, H. (1929) "Stability in Competition," *Economic Journal* 39, 41-57.
McKelvey, Richard (1976) "Intransitivities in Multidimensional Voting Models and Some Implications for Agenda Control," *Journal of Economic Theory* 12 : 472-82.
Palfrey, Thomas (1984) "Spatial Equilibrium with Entry" *Review of Economic Studies* 51: 139-156.
Plott, Charles (1967) "A Notion of Equilibrium and its Possibility Under Majority Rule" *American Economic Review* 57: 787-806.
Shepsle, Kenneth (1991) *Models of Multiparty Electoral Competition*, Harwood Academic Publishers.
Wittman, D. A. (1983) "Candidate motivation: a synthesis of alternatives," *American Political Science Review* 77, 142-157.

Candidate Quality and Electoral Competition: Theory and Data

Enriqueta Aragones
Thomas Palfrey

Candidates' Quality Dimension

- Competence
- Incumbency Effect
- Economic Performance
- Negative Campaigning
- Charisma
- Scandal
- Sex Appeal

<u>Candidate's Advantage:</u>

Any non-policy advantage equally valued by all voters.

A small modeling change ⇒ ⇒ Big difference in predictions

If
- candidates are only office motivated
- uncertainty about the median voter

Then
- weak candidates must differentiate to win
- strong candidates exploit opponents weaknesses
- ☑ POLICY DIVERGENCE

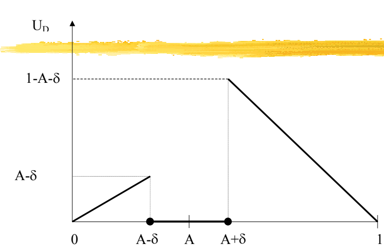
Other things can also lead to policy divergence

- multicandidate races; or
- multiple policy dimensions; or
- Policy motivated candidates (with uncertainty)
- ☑ POLICY DIVERGENCE

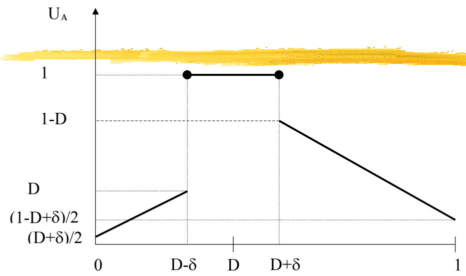
This is a fundamentally different reason
- weak candidates must differentiate to win
- strong candidates exploit opponents weaknesses

The model

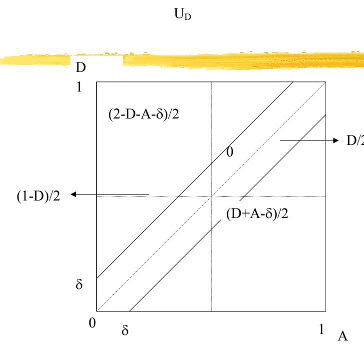
- Policy space $X \subseteq [0,1]$
- Candidates:
  - *A* and *D*
  - max probability of winning
- Voters:
  - $x_m \sim P(X)$
  - $U_i(x_A) = \delta - |x_i - x_A|$, $0 < \delta < 1/2$
  - $U_i(x_D) = -|x_i - x_D|$



Indifferent voters vote for candidate A.



Indifferent voters vote for candidate A.



Proposition:

- If $0 < \delta < 1/2$ there is no pure strategy Nash equilibrium.
- We look for equilibrium in symmetric mixed strategies.

Continuous Policy Space

$X = [0,1]$

$0 < \delta < 1/2$

$x_m \approx$ Uniform on X

Theorem:

There exists a mixed strategy equilibrium with symmetric strategies.

(Dasgupta and Maskin 1986)

Dasgupta and Maskin's Conditions:

- Strategy space is a closed interval.
- Payoff functions are continuous except in a set of measure zero.
- The sum of the payoffs is upper semi-continuous.
- Payoffs are bounded.
- Payoff functions are weakly lower semi-continuous.

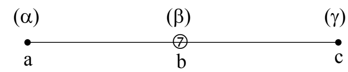
Equilibrium Strategies:

- lower bound of D's support = lower bound of A's support - δ
- for any $n > 1$ there is an $\varepsilon > 0$ such that lower bound of A's support $< 1/2 - n\,\delta$ for all $\delta < \varepsilon$
- A's lower bound $\to 1/2$ as $\delta \to 0$
- gap for D around 1/2
- no mass points at extremes of supports.
- $s^D(y)$ decreasing for $y < 1/2$

Equilibrium Payoffs:

- $EU_A \geq EU_D$
- $EU_A \to 1/2$ as $\delta \to 0$
- $EU_D \to 1/2$ as $\delta \to 0$

Example: $\alpha + \beta + \gamma = 1$



D

		a	b	c
A	a	1,0	$\alpha,1-\alpha$	$\alpha+\beta,1-\alpha-\beta$
	b	$1-\alpha,\alpha$	1,0	$\alpha+\beta,1-\alpha-\beta$
	c	$1-\alpha,\alpha$	$1-\alpha-\beta,\alpha+\beta$	1,0

Symmetry: $\beta=1-2\alpha$

- $s^A(a) \leq s^A(b)$
- $s^D(a) \geq s^D(b)$
- $\alpha\uparrow \Rightarrow s^A(a)\uparrow$
- $\alpha\uparrow \Rightarrow s^D(a)\downarrow$
- $\alpha=1/2 \Rightarrow s^*=(1/3,1/3,1/3),(1/3,1/3,1/3)$, D wins with 1/3
- $\alpha\to 0 \Rightarrow s^A(b)\to 1$, $s^D(b)\to 0$, D wins $\to 0$

Discrete Policy Space

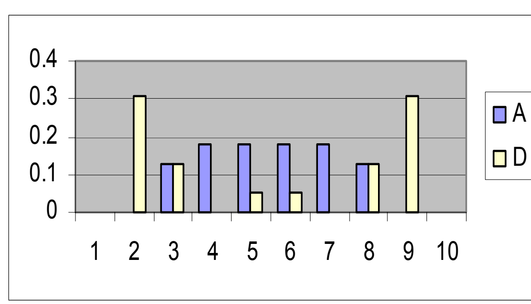
$X = \{x_i \in [0,1]: x_i=(i-1)/(n-1), i=1,2,\ldots n\}$
for $n > 1$
$0 < \delta < 1/(n-1)$
$x_m \approx$ Uniform on X

Proposition:

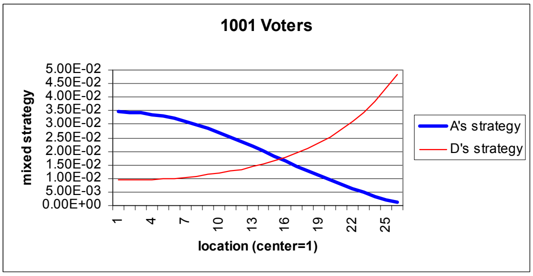
There exists a mixed strategy equilibrium with symmetric strategies and no gaps.

In any equilibrium with symmetric strategies and no gaps:

- D's mixing distribution is U-shaped with least probability weight in the center.
- A's mixing distribution places monotonically decreasing weight on strategies that are further from the center
- The end points of the supports are either the same or very close.
- $EU_A > 1/2 > EU_D$



Example with 1001 Voters

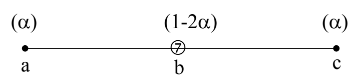


Limiting properties, n $\rightarrow \infty$

- Notice: $\delta \rightarrow 0$ as n $\rightarrow \infty$
- $EU_A \rightarrow 1/2$
- $EU_D \rightarrow 1/2$
- supports $\rightarrow 1/2$
- number of policies in the support $\rightarrow \infty$

☞ **Larger δ:** asymmetric strategies, gaps, ...

Experiment Design



	a	b	c
a	1,0	α,1−α	1-α,α
b	1−α,α	1,0	1-α,α
c	1−α,α	α,1-α	1,0

α=uncertainty index

3 levels of uncertainty

- $\alpha<1/3$ unimodal: (1/5,3/5,1/5)
- $\alpha=1/3$ uniform: (1/3,1/3,1/3)
- $\alpha>1/3$ bimodal: (3/7,1/7,3/7)

3 main hypothesis

- Statistical predictions (not point predictions)
- Quality Divergence Hypothesis (better candidates are more moderate)
- Polarization Hypothesis (more uncertainty, stronger divergence)

Protocol:

- Players: Red and Blue
- 100 times as A + 100(!) times as D
- Randomly matched opponents
- Last about 1,5 hours.
- Average payoff: $25
- Number of players: between 8 and 16

The three games:

UNIFORM

3,0	1,2	2,1
2,1	3,0	2,1
2,1	1,2	3,0

UNIMODAL

5,0	1,4	4,1
4,1	5,0	4,1
4,1	1,4	5,0

BIMODAL

7,0	3,4	4,3
4,3	7,0	4,3
4,3	3,4	7,0

The three “real” games:

UNIFORM

14,6	4,16	9,11
9,11	14,6	9,11
9,11	4,16	14,6

UNIMODAL

14,6	6,14	12,8
12,8	14,6	12,8
12,8	6,14	14,6

BIMODAL

14,6	6,14	8,12
8,12	14,6	8,12
8,12	6,14	14,6

The three games:

UNIFORM

3,0	1,2	2,1	14,6	4,16	9,11
2,1	3,0	2,1	9,11	14,6	9,11
2,1	1,2	3,0	9,11	4,16	14,6

UNIMODAL

5,0	1,4	4,1	14,6	6,14	12,8
4,1	5,0	4,1	12,8	14,6	12,8
4,1	1,4	5,0	12,8	6,14	14,6

BIMODAL

7,0	3,4	4,3	14,6	6,14	8,12
4,3	7,0	4,3	8,12	14,6	8,12
4,3	3,4	7,0	8,12	6,14	14,6

Data from Experiments:

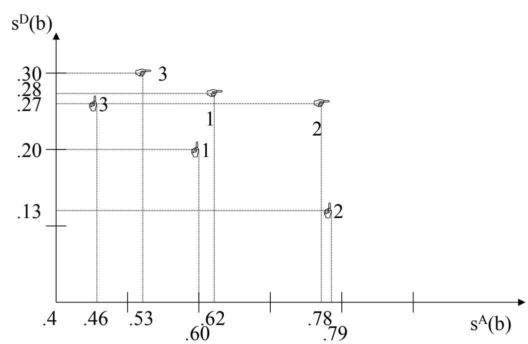
	p^*	q^*	p	q
Low α	.78	.11	.769	.252
N			4000	4000
Uniform	.6	.2	.609	.288
N			3400	3400
High α	.45	.27	.514	.320
N			3664	3664

Main features of the data

- The Quality Divergence Hypothesis is strongly supported by the data.
- The Polarization Hypothesis is strongly supported by the data.
- All signed comparative statics about p and q are statistically significant.
- A fits better the Nash predictions than D.
- Both tend to overplay center (strongest for D).

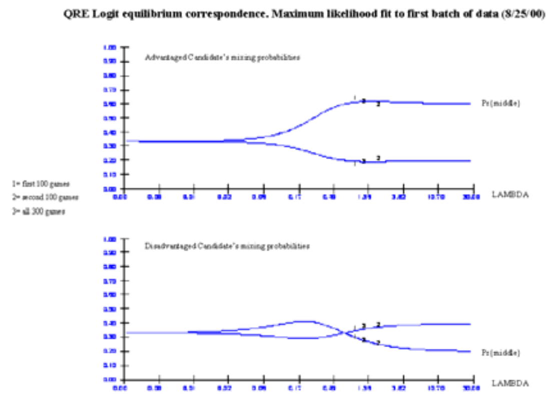
☝ **Equilibrium**

☞ **Data**

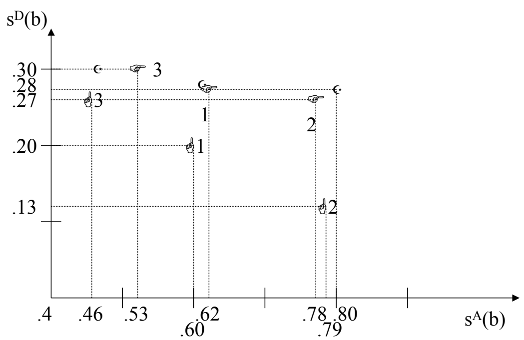


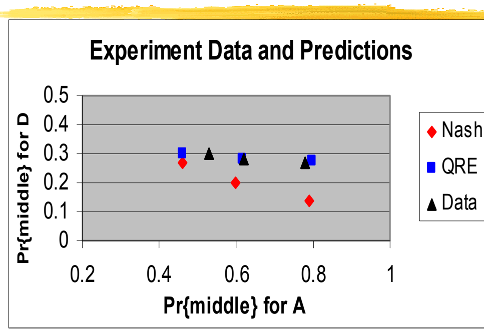
Expected Payoffs:

		a	b	c
uniform	A	9.5	10.4	9.3
	D	10.05	9.8	10.05
unimodal	A	11.12	12.59	11.19
	D	7.81	7.776	7.746
bimodal	A	9.5	9.8	9.5
	D	10.62	9.76	10.56



Equilibrium
Data
QRE Estimates





CONCLUSIONS and FURTHER DIRECTIONS

- Existence of Equilibrium
- Characteristics of Solution
  - Median voter theorem fails
  - A tends to middle
  - D tends to outside
  - A.S. Convergence to median as $\delta \to 0$
- Experimental support, with QRE adjustment of equilibrium
  - A converges quickly
  - D overplays middle
- Still working on solution with continuous policy space
- Other directions:
  - Parties as Quality Screening Mechanisms
  - Primaries as Beauty Contests
  - Role of pressure groups
  - Policy motivated candidates
  - Combine with campaign spending game--spend to enhance image