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A theory of participatory democracy based on the real case of Porto Alegre

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ABSTRACT

Participatory democracy is a process of collective decision making that combines elements from both direct and representative democracy: Citizens have the power to decide on policy proposals and politicians assume the role of policy implementation. The aim of this paper is to understand how participatory democracy operates, and to study its implications over the behavior of citizens and politicians and over the final policy outcomes. To this end, we explore a formal model inspired by the experience of Participatory Budgeting implemented in the Brazilian city of Porto Alegre, that builds on the model of meetings with costly participation by Osborne et al. [2000. Meetings with costly participation. American Economic Review 90, 927–943].

1. Introduction

> We are not selling the illusion of the direct democracy in the Greek plaza which, let us bear in mind, was not the democracy of the all but the democracy of the best.
>
> Olivio Dutra, first Workers Party mayor of Porto Alegre.

Participatory democracy is a process of collective decision making that combines elements from both direct and representative democracy: Citizens have the power to decide on policy proposals and politicians assume the role of policy implementation. The electorate can monitor politicians' performance simply by comparing citizens' proposals with the policies actually implemented. As a result, the discretion of politicians is severely constrained. In this system, the extent to which citizens can affect policy and determine social priorities is directly aligned with the degree to which they choose to involve themselves in the process.[1]

Real life experiences of participatory democracy have mainly materialized in processes of "Participatory Budgeting" at the city level. This is the case of nearly 200 Brazilian municipalities where direct democracy, in the form of popular assemblies, coexists with formal political parties and local elections: Citizens have to make a budget proposal but they also

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[1] Although inspired by earlier figures such as Rousseau or John Stuart Mill, the first theoretical formulations of participatory democracy were made during the 70s by Pateman (1970) and Macpherson (1977). An excellent discussion of the main features of this model of democracy can be found in Held (1987).

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have to elect the city executive and legislative bodies. Participatory systems have also been implemented at the state level in Rio Grande del Sul (Brazil) and in West Bengal and Kerala (India), and at the school level in Chicago, through the Local School Councils.[2]

The aim of this paper is twofold: First, we want to understand how participatory democracy operates and to study how it shapes the behavior of citizens and politicians and the final policy outcomes. To this end, we explore a formal model inspired by the experience of the Brazilian city of Porto Alegre, the most successful and lasting real example of Participatory Budgeting. This formal model combines classical elements of the models of electoral control with classical features of the models of voter participation. In particular, we consider two different types of voter participation in the political process: On the one hand citizens decide whether to attend a meeting that they know is relevant for the final policy choice and on the other hand, they have to decide whether or not to reelect the incumbent politician.

Our second goal is precisely to discuss the application of this system to the particular case of Porto Alegre. Electoral control is an issue of paramount importance in Brazil, where politics have been characterized for a long time by authoritarianism and corruption at the state level and clientelistic relationships between local politicians and popular classes. This led to a lack of legitimacy of political institutions and to a general discredit of government proposals that the constitution of 1988 tried to alleviate by starting a process of strong decentralization. Initiatives like Participatory Budgeting were aimed at making policies more responsive to the preferences of citizens and to eradicate the use of "gift exchanges" at the local level. The results of the formal analysis allow us to construct an explanation for the political events occurred in that city, in terms of citizen participation and control of politicians, during the period in which this system has been operating.

Participatory democracy and the experience of Porto Alegre must be of interest to economists for several reasons. On the one hand, Economics has exclusively focused on more traditional models of democracy like direct democracy and, above all, representative democracy, since the latter is the prevailing system in the Western world. But the success of the mentioned real life examples of participatory democracy on both economic and social grounds[3] demands an evaluation of its potential virtues and flaws. The present paper represents a first step in that direction. On the other hand, two facts occurred in Porto Alegre escape an obvious rational explanation: The substantial participation of citizens at meetings despite the explicit costs of attendance, and that the policies implemented have consistently followed the citizens' proposals. Our formal analysis sheds light on these issues. In particular, we characterize participation at the meetings in terms of preferences of the attendees and we offer a theoretical explanation for the increasing participation rates observed in Porto Alegre. We also offer an analysis of the government's incentives to fulfill the society's demands under this system.

1.1. Motivation: The real case of Porto Alegre

Porto Alegre is the capital of the Brazilian state of Rio Grande del Sul with a population of 1.4 million inhabitants. The city is characterized by an income slightly above the Brazilian average and a high number of politically involved community associations. Its system of Participatory Budgeting (*Orçamento Participativo*), referred to as OP, is the best known and most successful experiment of local management based on participatory democracy. It was introduced in 1989 when the Workers Party (*Partido dos Trabalhadores*, PT henceforth) won the local elections.[4]

The OP is a pyramidal system whose main elements are: The regional and thematic assemblies, the Fora of Delegates, and the Council of the OP (COP). Regional assemblies, called *rodadas*, take place in each of the 16 regions of the city. The assemblies are the principal forums for popular participation; they are totally open and any citizen may attend them. In these meetings, each region evaluates the executive's performance, defines its priorities and demands, and elects delegates for the Forum of Delegates and councillors for the COP. Prior to the *rodada,* preparatory meetings organized by the community take place.

Public scrutiny and control of the municipal government is one of the main issues at the meetings. The municipality accounts for the implementation of the previous year Investment Plan. After that, discussions focus on setting a consensual rank of priorities for each region and a list of hierarchical demands inside each priority. Each region selects as priorities five out of the 13 areas available.[5] All decisions are taken by majority rule.

The Fora of Delegates is formed by about 1000 delegates. Their role is to serve as intermediaries between the COP and the citizens. They supervise the implementation of the budget and inform the population. Delegates are typically leaders of community organizations, so citizens not integrated in these structures are hardly elected.

Finally, the COP is a body composed by councilors elected by the assemblies, the Residents Association Union of Porto Alegre, and the City Hall's Attendants Union. Its role is to design and submit to the city government a detailed budget proposal based on the priorities determined in the regional assemblies, and to monitor the execution of the approved public works.

[2] For a more detailed description of these cases, see Fung and Wright (2001).

[3] On the positive social and economic consequences of participatory democracy, see Fung and Wright (2001), Santos (1998) and the report on Participatory Budgeting of the Inter-American Development Bank (2005).

[4] The description of Participatory Budgeting in Porto Alegre builds on Santos (1998) and Marquetti (2000, 2003).

[5] The issue areas are: Basic Sanitation, Land use regulation (that includes Housing policy), Transportation, Social Service, Education, Health, Paving, City organization, Leisure, Sports, Culture, Environment and Economic Development.

The OP is an example of participatory democracy because it reconciles direct democracy, embedded in associations and meetings, and representative democracy at the urban level. OP coexists with two elected bodies who hold the formal municipal power: The Mayoralty or executive body (*Prefeitura*) and the Chamber of Deputies or legislative body (*Câmara de Vereadores*). The COP submits the budget proposal to the Chamber of Deputies who has total autonomy to amend or defeat it. However, since the proposal has been approved by citizens, assemblies, and community organizations, the political cost of turning it down is very high. But the relationship between the "formal" elected representatives and popular movements has not been without problems. In fact, the conflict between them has been one of the main political issues in Porto Alegre.

The OP has had three main effects on Porto Alegre. First, the city has witnessed a remarkable improvement of the behavior of politicians and community leaders who, as in the rest of Brazil, were used to clientelism. Now, the city councilors and potential candidates face a more informed population and grass-root organizations, so there is little room for the "gift exchange" that characterizes clientelism. The high degree of accountability of the administration has reduced corruption and rent-seeking behavior.

Another remarkable success of the OP has been the massive engagement of citizens in the process. The number of participants in the meetings has continuously increased even though in relative terms no more than the 5% of citizens are directly involved.[6] But most importantly, there has been a substantial participation of the less educated and wealthy segments of the population, precisely those typically disengaged from the political process prior to the adoption of the OP. This has led to the third effect, a very high level of redistribution of income within the city. These two effects will be discussed more in detail in relation to our model later in the paper.

1.2. Overview of the model

Our analysis builds on the model of pure direct democracy by Osborne et al. (2000). There, the members of a society decide independently whether to attend a meeting, at a cost, where the policy decision taken will be a compromise among the attendees' ideal positions. Attendance is based on a cost–benefit calculation: Citizens compare the cost of participation with the impact that their presence will have on the compromise. We extend their analysis by considering the existence of a representative or *legislator* who is in charge of policy implementation. The legislator can choose freely the policy to be implemented. We assume that the legislator has her own preferences over policies and she also cares about reelection.

Notice that the roles played here by citizens and representatives differ from the roles they play in a standard model of representative democracy. In our model of participatory democracy, citizens are the first ones to move by making a policy proposal, and representatives have to react to it, deciding whether to implement it or not. In a standard model of representative democracy the policy decision is made by the elected representatives and the electorate reacts to it, approving or disapproving the policy choice with their vote in future elections.

Even though the process of Participatory Budgeting is an annual process, citizens only reelect legislators every four years. This implies that every time that a legislator faces reelection voters evaluate her performance over four different policy decisions corresponding to four policies proposed by the assembly. Thus it is natural to assume that, when electing the legislator, citizens vote retrospectively[7]: They will tend to reelect the legislator if they approve of her average policy choice of the current legislature. Thus, the voting decision is unlikely to be based on the legislator's policy choice in any particular issue but rather on her average performance. In order to simplify the model, we consider a reduced form game in which one policy choice represents the whole set of decisions taken by a legislator during a legislature.

In particular, we assume that citizens reward those legislators that implement policies proposed by the citizens' assembly by reelecting them; and punish those who do not take the assembly's proposals into account by not reelecting them. This retrospective voting rule seems to fit well the behavior of citizens involved in a process of participatory democracy.[8] By agreeing on a policy proposal at the assembly, citizens resolve the conflict of interests due to the heterogeneity of their policy preferences. And, at the time of deciding on the reelection of the legislator, they only need to evaluate his performance by comparing the policies they proposed and the policy choices of the legislator. Thus, society's preferences with respect to the performance of the legislator may be considered to be homogeneous. Nevertheless, since we assume that both, citizens and the legislator, care about the policy implemented, the conflict between the legislator's and the citizens's preferences allows for the possibility of a variety of equilibria. Hence, citizens in this model punish the legislator if they do not approve of her performance by not coordinating their votes for her in future elections; otherwise

[6] The City Hall calculated that if the unofficial preparatory and intermediate meetings were considered, the number of citizens involved in the OP process in 1999 would be of around 100.000, representing the 8% of the voting age population. See Section 5.

[7] There exists some theories of voting which suggest that voters base their decision on past performance of parties. This literature starts with Downs' (1957) theory according to which, parties' past performance is the cheapest way for voters to predict future performance. It continues with the reward-punishment theory proposed by Key (1966), and with the empirical studies by Fiorina (1981), among others. Other models and applications of retrospective voting include: Barro (1973), Ferejohn (1986), Alesina and Rosenthal (1989), Austen-Smith and Banks (1989), and Persson and Tabellini (2000).

[8] For instance, in Porto Alegre there are committees formed by elected delegates whose function is to supervise the implementation of the budget. Since they have the right to ask the city government for detailed explanations on each investment work, any deviation that cannot be explained by sound technical or economic criteria is liable to have negative electoral consequences.

they reward the legislator by reelecting her. This assumption captures the high degree of accountability in participatory democracy mentioned above.[9]

More specifically, we model the system of participatory democracy as a game in three stages. In the first stage, each citizen decides whether to attend or not to attend a meeting in which a policy proposal will be decided. In the second stage, citizens who attend the meeting come out with a policy representing their interests and their delegates make a proposal to the legislator, aimed at making her implement the assembly's choice. In the third stage, the legislator decides the policy to be implemented, taking into account the reduced form game that includes her chances of reelection.

1.3. Overview of the results

Following the intuition of the retrospective voting models, our first result shows that the set of policies that can be implemented in equilibrium is a subset of the policy space that contains the ideal point of the legislator. That is, the legislator will only implement policies that are close to her ideal point up to the extend that the legislator is indifferent between jeopardizing her reelection by implementing her ideal policy, or guaranteeing her reelection by satisfying the assembly. We define this policy location the maximal compromise policy. We show that the more the legislator cares about holding office the larger is the set of policies that can be implemented in equilibrium. The intuition is clear: A legislator who does not care so much about policy is willing to accept proposals further apart from her ideal point in order to guarantee a sure win in a future election. On the other hand, the softer is the threat of punishment, the smaller the size of the set of implementable policies. That is, a legislator who believes that her chances of being reelected will be very low unless she follows the policy proposed by the assembly, will be willing to implement a larger set of policies.

The results we obtain show the relevance of two features: (1) The *alignment* between the policy preferences of the legislator and the policy preferences of society; and (2) the degree of *extremism* of the legislator. Formally, we find that pure strategy equilibria can only be of two types: Interior equilibrium or maximal compromise equilibrium.

Interior equilibrium: If the most preferred outcome to the society lies relatively close to the legislator's ideal point, that is, when society and legislator's preferences are aligned, the policy implemented in equilibrium is the assembly's most preferred policy. This equilibrium exists only for a certain distributions of preferences. Constraints on these distributions are stronger when society holds extremist views. Still, this result implies that extremist policies can be implemented in equilibrium provided that alignment occurs.

Maximal compromise equilibrium: If the legislator is extremist in the political spectrum, for any distribution of preferences, there is an equilibrium in which she implements the maximal compromise policy. This implies that when politicians are extremist, society will tend to act as a countervailing power and moderate her.

The rest of the paper proceeds as follows. The next section describes the formal model. Section 3 presents the sequential derivation of optimal policy choices. Section 4 analyzes the citizens' choice on participation in the assemblies. Section 5 analyzes the evidence from Porto Alegre in the light of the results obtained in the previous sections. Finally, Section 6 contains some concluding remarks. All proofs can be found in Appendix A.

2. The model

The policy space is continuous and one-dimensional, and represented by the interval $[0,1]$. There is a finite number N of citizens with single-peak preferences over the interval $[0,1]$. The citizens' ideal points are distributed according to a probability distribution $F(\theta)$ with support in $[0,1]$. We will assume, without loss of generality, that there are not two citizens with the same ideal point and that there is always at least one citizen with ideal point $\theta_i = 0$ and at least one citizen with ideal point $\theta_i = 1$.

At the first stage of the game, citizens decide whether to attend a meeting in which a policy will be proposed. Attendance implies that their opinion will be taken into account in the elaboration of that policy proposal, but it also involves an individual cost $0<c<\frac{1}{2}$. This may include transportation costs or the opportunity cost of the time spent in the assembly. The welfare of a citizen i with ideal point θ_i depends on the policy implemented and on whether he attends the meeting and it is given by the following expression:

$$V_i(x_L, a_i) = -|x_L - \theta_i| - a_i c,$$

where x_L is the policy implemented, and a_i represents the decision of citizen i on whether to attend the meeting: If $a_i = 1$, i attends the meeting and pays a cost c, if $a_i = 0$, i does not attend the meeting and pays no cost.

We assume that the citizens who attend the meeting aggregate their preferences according to some previously fixed aggregation rule and that the policy selected by that rule is the one that the assembly would like to see implemented. Let X denote the list of ideal points of those citizens who attend the meeting, $|X|$ the number of attendees, and let $\theta_A(X)$ denote

[9] There is a relationship between the present model and the recent developments in the theory of political agency, but important differences remain. First, these papers often study voters' decision problems from a moral hazard or an adverse selection perspective (see Ferejohn, 1986; Besley and Case, 1995, respectively) or by combining both (Banks and Sundaram, 1998). On the contrary, in our model voters can perfectly monitor the legislator actions. But beyond this, the retention rule in this literature is often imposed, as in Ferejohn (1986), and always set costlessly. In our case, although a rule is given, its actual enforcement depends upon the citizens costly participation in the meeting.

the assembly's most preferred policy. The aggregation rule we consider selects the ideal point of the median of the set of attendees whenever it is unique. Otherwise, when there are two medians in the set of attendees, the aggregation rule selects the middle point between the two.[10]

Given a distribution of ideal points of the citizens, $F(\theta)$, let θ_m denote society's most preferred policy defined according to the corresponding aggregation rule. That is, θ_m would be the assembly's decision if all individuals participate in the assembly. Notice that the policy chosen by the assembly at the meeting depends on the distribution of ideal points of the citizens who decide to attend the meeting, X, and only indirectly it depends on $F(\theta)$.

After the assembly takes place, a proposal is transmitted to a *legislator* who is in charge of implementing the final policy. In Porto Alegre, two bodies formed by delegates elected at the assemblies are in charge of doing this. We assume the existence of an intermediate body of *delegates* that construct a policy proposal to be presented to the legislator. We denote this policy proposal by x_A. We assume that the delegates' utility function coincides with the assembly's one so they try to induce the legislator to implement a policy as close as possible to θ_A.[11] This implies that the policy proposed by the assembly to the legislator, x_A, does not need to coincide with the assembly's most preferred policy θ_A; it is a strategic choice that takes into account the distortion introduced by the legislator.

The welfare of the legislator depends on her own policy preferences and on the probability of being reelected, and it is represented by a convex combination as follows:

$$V_L(x_A, x_L) = (1-\alpha)P(x_A, x_L) - \alpha|x_L - \theta_L|,$$

where $\theta_L \in [0,1]$ is the ideal point of the legislator, $\alpha \in [0,1]$ is an exogenous parameter that represents the intensity of the policy preferences of the legislator relative to her preferences for holding office. From the point of view of the legislator, $P(x_A, x_L)$ is interpreted as the probability with which she will be reelected. It depends on the amount of support that she will be able to obtain from the population, which in turn depends on whether the citizens approve her performance. As mentioned, citizens judge the legislator' performance according to how close her choice x_L is from the general will represented by their proposal to her, x_A. In particular, we assume that the probability of reelection $P(x_A, x_L)$ is a step function of the distance between the implemented policy x_L and the mandate of the citizens' assembly x_A:

$$P(x_A, x_L) = \begin{cases} 1 & \text{if } |x_L - x_A| \leqslant B, \\ \varepsilon & \text{otherwise,} \end{cases}$$

where $B>0$ is the degree of discretion of the legislator, that may account for financial or technical circumstances unforeseen by the citizens. In participatory democracy, legislators still control this knowledge and have privileged access to it.[12] So if the difference between the policy proposed by the delegates and the policy implemented by the legislator is not larger than this degree of discretion B, citizens will approve the legislator's performance and they will likely reelect her in future elections. Otherwise, the reelection of the legislator is compromised and we assume that she will only win future elections with probability ε, with $0<\varepsilon<1$. We assume that ε takes small values, reflecting the high degree of accountability of the participatory process.[13] Finally, if nobody attends the meeting the legislator can implement her ideal point and she is reelected with probability one.

Our assumption of common preferences on the legislator's performance can alternatively be interpreted as an implicit commitment on the part of the majority of citizens to punish legislators who do not take their proposal into account: If the legislator deviates more than B, this deviation will be socially regarded as a reason to vote her out. This is partially motivated by the case of Porto Alegre where citizens are concerned about a return to authoritarian politics and they organize rallies, meet legislators and follow very closely the debates when the budget proposal is passed to the city legislative body. Notice that this rule only defines the reelection probabilities, thus if the assembly approves the legislator's performance, the reelection occurs with probability one, which only implies that it is necessary that the median voter approves the performance. The equilibrium results will show that this is the case, and therefore this function is consistent with the rest of the behavioral assumptions of the model.

So far, we have constructed a game in three stages. In the first stage, citizens decide whether to attend or not to attend the meeting. In the second stage, citizens who attend the meeting choose a policy and the delegates make a proposal to the legislator. In the third stage, the legislator decides which policy will be finally implemented.

[10] We are of course aware of the fact that with two medians the choice of this policy cannot be rationalized by a voting process within the assembly. However, this assumption allows us to pin down a unique compromise regardless of the number of participants.

[11] The strength of the bond between the delegates and the regions they represent has been a source of unrest in Porto Alegre: CIDADE, a NGO monitoring the process, has reported discussions about delegates who allegedly took positions without consulting their constituencies or who failed to report back decisions at higher levels. Nevertheless, the strong accountability and the reelection rules employed have tempered the potential for corruption. Surveys show that over the 85% of the population thinks that delegates "always" or "most of the time" represent the assemblies' interests (CIDADE, 2002).

[12] In Porto Alegre, the executive has been accused of abusing its privileged position when resorting to "technical reasons" in order to challenge the budget proposal. This problem was serious enough to prompt the COP to start training seminars for delegates and councillors.

[13] The discrete form for $P(x_A, x_L)$ can be dropped without qualitatively changing the results. Section 6 contains a discussion on the effects of employing continuous functional forms.

3. Optimal policy choices

In order to find the subgame perfect equilibrium strategies of this game, we analyze in this section the optimal choices of the players by backward induction, starting with the optimal reaction of the legislator, in terms of policy choices, to a given proposal made by the delegates. Then, we analyze the optimal proposal of the assembly's delegates for a given distribution of preferences of the attendees, taking into account the optimal reaction of the legislator. Finally, we characterize the equilibrium policy choice for any pair of legislator's and assembly's policy preferences, (θ_L, θ_A).

3.1. The optimal choice of the legislator

In order to choose the policy that will be finally implemented, x_L, the legislator maximizes her payoff, given a policy proposed by the delegates, x_A:

$$\max_{x_L} V_L(x_A, x_L) = (1-\alpha)P(x_A, x_L) - \alpha|x_L - \theta_L|,$$

where

$$P(x_A, x_L) = \begin{cases} 1 & \text{if } |x_L - x_A| \leqslant B, \\ \varepsilon & \text{otherwise.} \end{cases}$$

Let us define $b = ((1-\alpha)/\alpha)(1-\varepsilon)$. Notice that $(1-\varepsilon)$ represents the probability with which the legislator is not reelected when she deviates too much from x_A, and $(1-\alpha)/\alpha$ represents the value of holding office. Thus, b represents a measure of the cost that the legislator has to pay when she is punished by the electorate. Straightforward computations allow us to characterize the best response of the legislator. That is,

$$x_L(x_A) = \begin{cases} x_A + B & \text{if } B \leqslant |x_A - \theta_L| \leqslant b + B \text{ and } x_A \leqslant \theta_L, \\ x_A - B & \text{if } B \leqslant |x_A - \theta_L| \leqslant b + B \text{ and } x_A \geqslant \theta_L, \\ \theta_L & \text{otherwise.} \end{cases}$$

The intuition behind this function is easy to grasp: Consider first the case in which the delegates' proposal is very close to the legislator's ideal point, i.e. $|x_A - \theta_L| < B$. In that case, the legislator can implement her ideal point without compromising her reelection. Similarly, when $|x_A - \theta_L| > b + B$, the delegates' proposal is so far away from the legislator's ideal point, that she prefers to ignore the proposal even though that implies jeopardizing her reelection. Only in the remaining case the proposal of the delegates is so far from the legislator's ideal point that she cannot implement her ideal point without compromising her reelection, but it is close enough so that the legislator prefers to compromise and guarantee a sure victory in the election. In this case the optimal choice of the legislator is a policy that lies between the assembly's proposal and her own ideal point such that it is exactly a distance B away from the assembly's proposal.

3.2. The optimal choice of the delegates

The policy proposal x_A is made by a small group of delegates. They are committed to induce the legislator to implement a policy as close as possible to θ_A and they are aware of her preferences. The optimal policy choice of the delegates can be then characterized by the following function:

$$x_A(\theta_A) = \begin{cases} \theta_L - b - B & \text{if } \theta_A \leqslant \theta_L - b, \\ \theta_A - B & \text{if } \theta_L - b \leqslant \theta_A \leqslant \theta_L, \\ \theta_A + B & \text{if } \theta_L \leqslant \theta_A \leqslant \theta_L + b, \\ \theta_L + b + B & \text{if } \theta_L + b \leqslant \theta_A. \end{cases}$$

In the second and third cases, the assembly's most preferred policy is close to the legislator's ideal point, i.e. $|\theta_L - \theta_A| \leqslant b$, and the delegates can induce the legislator to implement the assembly's most preferred policy by making proposals B away from θ_A.[14] Otherwise, the best the delegates can do is to induce a compromise. The will propose a policy that makes the legislator indifferent between implementing her ideal point and jeopardizing the next election, and implementing the maximal compromise policy that still assures her approval, that is, a policy of maximal compromise.

3.3. The equilibrium policy choice

By combining the optimal choices of delegates and legislator we can characterize the policies that will be implemented in equilibrium as a function of the legislator's ideal point and the most preferred policy of the assembly. The next

[14] Note that the role of B ultimately boils down to the level of distortion that citizens introduce in their proposals. Therefore, it might be set to zero.

proposition presents the most immediate consequence of introducing a legislator's choice in a process of pure direct democracy: Not all policies are implementable in equilibrium.

Proposition 1. *In equilibrium the implemented policy x_L lies in the interval* $[\theta_L - b, \theta_L + b]$.

All proofs can be found in Appendix A.

This result could be seen as equivalent to imposing an exogenous restriction on the policy space; and it is so, strictly speaking. But these restrictions are not arbitrary. As in the retrospective voting models, they come from the legislator's preferences over policies and reelection. Note that the size of the set of implementable policies depends positively on $b = ((1-\alpha)/\alpha)(1-\varepsilon)$. And this is in turn decreasing in the value that the legislator attaches to holding office (α) and in the probability with which the legislator is reelected when the assembly feels deceived (ε).

These relations are intuitive: Legislators who do not care so much about policy are willing to accept proposals further from their ideal point in order to stay in office. By the same token, a legislator who believes that her chances of being reelected will be very low unless she follows the policy proposed by the assembly, will be willing to compromise.

Finally, the combination of all the previous derivations and assumptions allow us to derive a function mapping any pair of legislator's and assembly's policy preferences (θ_L, θ_A) into a unique policy outcome x_L[15]:

$$x_L(\theta_A, \theta_L) = \begin{cases} \theta_A & \text{if } \theta_A \in [\theta_L - b, \theta_L + b], \\ \theta_L - b & \text{if } \theta_A < \theta_L - b, \\ \theta_L + b & \text{if } \theta_A > \theta_L + b. \end{cases}$$

When the policy most preferred by the assembly is relatively close to the legislator's ideal point, the policy finally implemented coincides exactly with the preferences of the assembly. Otherwise, citizens cannot induce the legislator to implement their most preferred policy. They can at most induce a compromise between the policy preferences of the assembly and the legislator's. Moreover, the less the legislator cares about policy and the smaller the probability with which she is reelected when the assembly feels deceived, the larger the chances that the assembly can achieve its most preferred policy. In fact, if the values of α and ε are sufficiently small (so that b is large enough) the set of implementable policies may be the whole policy space. Therefore, if citizens could commit to a certain degree of punishment, in terms of the probability represented by ε, their optimal choice should be $\varepsilon = 0$, the maximal degree of accountability.

4. Endogenous participation

In this section, we analyze citizens' attendance decisions, taking as given the equilibrium policy choice derived above that embeds the optimal play of all agents in the continuation of the game.

In this subgame, an equilibrium is a list of values for $\{a_1, \ldots, a_N\}$ with $a_i \in \{0, 1\}$ such that for all i, a_i is a best response against $\{a_1, \ldots, a_{i-1}, a_{i+1}, \ldots, a_N\}$. We show that when the cost of attendance is large enough, relative to the parameters of the objective function of the legislator, there is a unique equilibrium in which nobody attends the meeting.

Proposition 2 (*Non-attendance*). *If $c > b$ there is a unique equilibrium in which nobody attends the meeting.*

As in Osborne et al. (2000) citizens perform a cost–benefit analysis in order to decide whether to attend the meeting. If the cost of attending is larger than the benefit they will obtain from the impact that their presence at the meeting will have on the final policy, they decide not to attend. We have assumed that if nobody attends the meeting the legislator can implement her ideal point without compromising her future reelection. Thus, in this model the size of b represents the maximal effect that any citizen can have on the policy implemented. Clearly, when the cost of attendance is larger than b nobody has any incentive to attend the meeting.

Next we show that when this is not the case, that is, if the cost of attendance is small enough, in equilibrium there must be some positive attendance.

Proposition 3. *If $c < b$, then no-attendance is not an equilibrium.*

In fact, the proof of Proposition 3 shows not only that nobody attending is not an equilibrium, but also that if nobody attends the meeting any citizen whose ideal point is more than a distance c away from the legislator's ideal point would be better off attending. The intuition of this result comes again from the cost–benefit analysis that citizens perform: Any citizen who attends the meeting can induce the legislator to implement either the citizen's ideal point or a maximal compromise policy (a policy that is a distance b away from the legislator's ideal point). In both cases, if the ideal point of the citizen is more than a distance c away from θ_L, the benefit for the citizen is larger than c, which is the cost of attending the meeting.[16]

Next we characterize some equilibrium strategies when $c < b$, that is, when there are some citizens who attend the meeting in equilibrium.

[15] The derivation of this function can be found in the proof of Proposition 1.

[16] When $c = b$ some citizens (or all, depending on the distribution of ideal points of the population) could be indifferent between attending and not attending the meeting when nobody else is attending. Existence of equilibrium in this case would depend on how indifferences are resolved.

4.1. Aligned moderate legislators

Let us assume that the distribution of ideal points of society, $F(\theta)$, is symmetric around the legislator's ideal point. This case will allow us to illustrate the cost–benefit analysis citizens have to perform when deciding whether to attend the meeting. In this case, the legislator's policy preferences qualify as moderate and aligned with society's. Notice that when we assume a distribution of citizens' ideal points that is symmetric around θ_L, we must have that $\theta_L = \frac{1}{2}$ given that we have assumed that there is at least one citizen with ideal point $\theta_i = 0$ and at least one citizen with ideal point $\theta_i = 1$.

Given the symmetry assumed in the structure of the game, we will find equilibria with symmetric distributions of attendees, that is, an equilibrium strategy profile $\{a_1, \ldots, a_N\}$ such that the distribution of the ideal points of the attendees is symmetric around θ_A. Since we must have $\theta_L = \frac{1}{2}$, in this equilibrium the policy preferred by the assembly coincides with the ideal point of the legislator, with the implemented policy, and with the most preferred policy of the society, i.e. $\theta_A = \theta_L = x_L = \theta_m = \frac{1}{2}$.

Next, we show that when the cost of attendance is relatively small, and the distribution of the ideal points of the citizens is symmetric around θ_L we can completely characterize equilibrium strategies in this case.

Proposition 4 (*Equilibrium with a symmetric distribution*). *For any finite number of citizens with distribution of ideal points symmetric around $\theta_L = \frac{1}{2}$, if $c < b$, there is an equilibrium in which the set of attendees $X = \{\theta_i : |\theta_L - \theta_i| > c\}$ and the policy implemented $x_L = \frac{1}{2}$.*

On the one hand, citizens who are close to the legislator's ideal point ($|\theta_L - \theta_i| < c$) become the median if they were to attend, but their impact on the policy outcome would be less than c. Hence, they prefer to stay home. On the other hand, citizens who are far away from the legislator's ideal point lose more than c from withdrawing. Thus, in this equilibrium all citizens whose ideal points are more of a distance c away from the legislator's ideal point will attend the meeting.[17] Note that in this extremely symmetric scenario, our results become a particular case of the results obtained by Osborne et al. (2000) our legislator's ideal point is equivalent to the default policy of their model.

4.2. Extreme legislators

Next we analyze the attendance equilibrium strategies when the ideal point of the legislator takes values other than $\frac{1}{2}$ and the distribution of society's preferences is not necessarily symmetric. In this case, two factors arise as critical: *How aligned* the legislator is with society's preferences and *how extremist* (in the spatial sense) she is with respect to $\frac{1}{2}$.

First we analyze the case in which the legislator is relatively extremist, that is, $|\theta_L - \frac{1}{2}| > b - c$. Recall that we assume $c < \frac{1}{2}$. In this case we find that for any distribution of citizens' ideal points there is an equilibrium in which the legislator implements the maximal compromise policy on her left if her ideal point is to the right of $\frac{1}{2}$, and she implements the maximal compromise policy on her right if her ideal point is to the left of $\frac{1}{2}$. Furthermore, only one citizen attends the meeting in this equilibrium: A leftist one if the legislator is rightist, and a rightist one if the legislator is leftist.

Proposition 5 (*Maximal compromise equilibrium*). *If $c < b$*:

(a) *The policy $x_L = \theta_L + b$ is an equilibrium outcome if and only if $\theta_L < \frac{1}{2} - (b - c)$. Moreover, only one citizen with ideal policy $\theta_i > \max\{\theta_L + b, 2(\theta_L + b - c)\}$ attends.*
(b) *The policy $x_L = \theta_L - b$ is an equilibrium outcome if and only if $\theta_L > \frac{1}{2} + b - c$. Moreover, only one citizen with ideal policy $\theta_i < \min\{\theta_L - b, 2(\theta_L - b + c) - 1\}$ attends.*

In order to grasp the intuition of this result, suppose that all citizens are located at either 0 or 1 and assume that the legislator's ideal point is extreme and closer to 0 than to 1. Any citizen with ideal point in 1 can force the legislator to implement a maximal compromise policy. This is an equilibrium because an additional citizen with ideal point at 1 cannot improve on this result and any citizen at 0 cannot change it.

Observe that the result stated in Proposition 5 holds for any distribution of the citizens' ideal points. When the legislator is extremist, citizens far from the legislator's ideal point have a strong incentive to participate. Moreover, one citizen is enough to induce the maximal compromise. But in this particular case, citizens who are extremist *and* close to the legislator prefer not to attend since that maximum compromise is not far enough from them.

In this equilibrium only one citizen attends the meeting, however, this result should not be taken literally. A real life interpretation of each citizen as defined in our model could be a community association. In fact, today about 600

[17] If there was $\theta_i = \frac{1}{2} - c$ and $\theta_{i'} = \frac{1}{2} + c$, in a symmetric equilibrium they both would have a weak preference to attend, and a weak preference not to attend, in case both are attending and also in case only one of them attends. Furthermore, if they both would attend, then in equilibrium all $\theta_j \notin (\frac{1}{2} - c, \frac{1}{2} + c)$ would have a weak preference for attending. If they both would not attend, then all $\theta_j \notin (\frac{1}{2} - c, \frac{1}{2} + c)$ would have a strong preference for attending. Thus, in this case there is also an equilibrium in which only $\theta_i = \frac{1}{2} - c$ and $\theta_{i'} = \frac{1}{2} + c$ attend.

community associations are established and active in Porto Alegre,[18] and the members of these associations account for a large proportion of the participation in the popular assemblies. Since it makes sense to identify each such community association with a particular position in the policy space, we can interpret our extreme result on attendance as an equilibrium in which only members of one community association attend the meeting. It still refers to a level of participation lower than in the interior equilibrium but it is not an implausible prediction: It does not refer to one individual having a large impact on the legislator's choice but to the members of an organized group forcing the legislator to a maximal compromise policy.

With extreme legislators we have thus an equilibrium in which only one citizen or association attends the meeting. This result can be thought of as an extreme case of the *low participation* result in Osborne et al. (2000). But it also challenges their *non-participation of the moderates* result: If the legislator preferences are too extreme, citizens at the center of the political spectrum have strong incentives to participate and moreover they can force her to implement the maximal compromise outcome in their favor. Note that this type of equilibria arises even if the legislator's policy preferences are aligned with the view of a majority of society.

4.3. Moderate legislators, skewed populations

The previous result also implies that when the legislator's policy preferences are not extreme in the spatial sense, a maximal compromise policy is never implemented in equilibrium.

Corollary 1 (*Moderate legislator*). *If* $|\theta_L - \frac{1}{2}| \leqslant b - c$, *in equilibrium we must have* $x_L \in (\theta_L - b, \theta_L + b)$ *and therefore* $x_L(\theta_A, \theta_L) = \theta_A$.

If the legislator is relatively moderate, in equilibrium the choice of the legislator must be a policy in the interior of the set of implementable policies. And given the optimal play of the delegates and the legislator in the continuation of the game, we know that the latter will implement the assembly's ideal policy.

Let us define an interior equilibrium as an equilibrium where $x_L \in (\theta_L - b, \theta_L + b)$. Next we show that in any interior equilibrium there will be two sets of ideologically extreme attendees divided by one set of non-attendees.

Proposition 6 (*Non-participation of the represented*). *If* $c < b$ *and* $x_L \in (\theta_L - b, \theta_L + b)$ *is the equilibrium outcome, then i attends if and only if* $|\theta_i - x_L| > c$.

In order to prove this result we first show that in this equilibrium any citizen not attending the meeting will leave the same number of attendees to his right and to his left. Then we show that the set of non-attendees is connected. Hence, the most preferred policy of the assembly, according to our rule, will always be the average of the two attendees surrounding the set of non-attendees. Let $\theta_l = \max\{\theta_i \in X : \theta_i < \theta_A(X)\}$ denote the most moderate leftist attendee, and let $\theta_r = \min\{\theta_i \in X : \theta_i > \theta_A(X)\}$ denote the most moderate rightist attendee. Then, the most preferred policy by the assembly is given by $\theta_A(X) = (\theta_l + \theta_r)/2$.

Proposition 6 shows that since incentives to attend are given by the impact of the attendance decisions on the final outcome, citizens close to the legislator prefer to stay home. Thus, we generalize the result on attendance provided by Osborne et al. (2000). Note that this proposition also generalizes Proposition 4 to values of the legislator's ideal point different from $\frac{1}{2}$, and to any distribution of the citizens' policy preferences.

Observe that this result holds for any value of the legislator's ideal point. Therefore, the interior equilibrium described here may potentially exist for any location of the legislator's ideal point. These results offer a broader picture of the *non-participation of the moderates* result in Osborne et al. (2000): Those citizens who are already represented by the equilibrium outcome or by the legislator's ideology, have no incentive to participate and will not attend the meeting. But they are not necessarily moderate in the political spectrum. Interestingly enough, this is consistent with the observed lack of participation of unions in the Porto Alegre's Participatory Budgeting process. Marquetti (2000) argues that one of the reasons explaining this is that unionists feel that they are already represented in the Participatory Budgeting by the leftist leaning of the municipal administration.

Observe that the conditions to be satisfied in an interior equilibrium are rather strong:

(i) There must be an identical number of attendees on both sides of the set of non-attendees, so $\theta_A(X) = (\theta_l + \theta_r)/2$.
(ii) All attendees must be at more than a distance c of the policy outcome.

What these two conditions imply is that the existence of an interior equilibrium will depend largely on the shape of the distribution of the ideal points of the population. Furthermore, they imply stronger restrictions on the set of implementable policies.

Corollary 2 (*Equilibrium policies*). *If* $c < b$ *and* $x_L \in (\theta_L - b, \theta_L + b)$ *is an equilibrium outcome, then* $x_L \in (\theta_L - b + c, \theta_L + b - c)$.

[18] The high number of politically active community associations is also one of the features that explains the introduction of the system of Participatory Budgeting in Brazil.

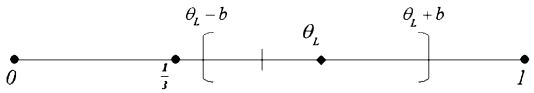

Fig. 1. Non-existence of equilibrium.

Next we show that a necessary condition for existence of an interior equilibrium for any value of the legislator's ideal point is that the policy preferences of the legislator must be aligned with those of society.

Corollary 3 (*Alignment for existence*). *If $c<b$ and $\theta_m \notin (\theta_L - b, \theta_L + b)$ then there is no interior equilibrium.*

With moderate legislators, if society's most preferred policy θ_m does not belong to the set of implementable policies then an interior equilibrium does not exist. Therefore, the alignment between the legislator's and society's preferences is a necessary condition for the existence of an equilibrium in pure strategies. The following example illustrates the non-existence of equilibrium in pure strategies when the median of society is not well aligned with the ideal point of the legislator and the legislator is moderate.

Example (*Non-existence of equilibrium*). Consider a society with only three citizens located at 0, $\frac{1}{3}$, and 1. Suppose that the legislator is located at $\frac{3}{5}$ and suppose that $b=\frac{1}{5}$. In this case, the ideal point of the median cannot be implemented in equilibrium. Suppose that the cost of attending is $c<\frac{1}{10}$. We will show that there is no pure strategy equilibrium (Fig. 1).

It is clear that all citizens want to attend when nobody is attending, since the attendance cost c is smaller than the impact each one of them has on the final outcome. If all of them were attending, the median of the assembly would be $\frac{1}{3}$, and the final outcome $\theta_L - b = \frac{2}{5}$. Since the legislator will not compromise beyond that, the citizen located at 1 would be better off withdrawing. If the only attendees were the two leftist citizens, any of them would be better off not attending. If the two extremist citizens were the ones attending the meeting, then the citizen at $\frac{1}{3}$ would prefer to attend since his utility would increase by $\frac{1}{2} - (\theta_L - b) = \frac{1}{2} - \frac{2}{5} = \frac{1}{10} > c$. If $\frac{1}{3}$ and 1 were the ones to attend, the citizen at 0 would prefer to attend since his utility would increase by $\frac{2}{3} - (\theta_L - b) = \frac{2}{3} - \frac{2}{5} = \frac{4}{15} > \frac{1}{10} > c$. Finally, if only one citizen attends the meeting and it is either 0 or $\frac{1}{3}$, then citizen 1 also wants to attend because his utility would increase by $\frac{1}{2} - (\theta_L - b) = \frac{1}{2} - \frac{2}{5} = \frac{1}{10} > c$ or $\frac{2}{3} - (\theta_L - b) = \frac{2}{3} - \frac{2}{5} = \frac{4}{15} > \frac{1}{10} > c$, respectively. And if the only citizen that attends is 1, then both citizens 0 and $\frac{1}{3}$ would like to attend because their utility would increase by $\frac{1}{2} - (\theta_L - b) = \frac{1}{2} - \frac{2}{5} = \frac{1}{10} > c$ or $\frac{2}{3} - (\theta_L - b) = \frac{2}{3} - \frac{2}{5} = \frac{4}{15} > \frac{1}{10} > c$, respectively. Hence, there is no equilibrium in pure strategies.

In Corollaries 1 and 2, we have shown that under certain conditions the system of participatory democracy ensures that society obtains its most preferred policy in equilibrium *if* it exists. However, Corollary 3 implies that otherwise we cannot make a clear prediction. Next we characterize general conditions on the distribution of citizens' ideal points that provide the necessary alignment to guarantee the existence of an interior equilibrium and therefore the implementation of society's most preferred policy.

Let us define the set

$$S = \left\{ x_L \in (\theta_L - b, \theta_L + b) : x_L = \frac{\theta_l(x_L) + \theta_r(x_L)}{2} \text{ for some } \theta_l(x_L) \text{ and } \theta_r(x_L) \right.$$
$$\left. \text{s.t. } \theta_l(x_L), \theta_r(x_L) \in (\theta_L - b, \theta_L + b) \text{ and } |\{\theta_i : \theta_i \leqslant \theta_l(x_L)\}| = \{\theta_i : \theta_i \geqslant \theta_r(x_L)\}| \right\}.$$

From previous results we know that an equilibrium outcome must belong to the set of implementable policies, $[\theta_L - b, \theta_L + b]$, and that an interior equilibrium outcome has to be the average of the ideal points of the two attendees delimiting the set of non-attendees. Thus, the policies contained in S are all candidates to equilibrium outcomes. Notice that if $\theta_m \notin (\theta_L - b, \theta_L + b)$ then the set S is empty. The next proposition derives a sufficient condition under which the elements of S can be implemented in equilibrium.

Proposition 7 (*Interior equilibrium*). *If $c<b$, then for each policy $x_L \in S$ there exist $\underline{c}(x_L)$ and $\overline{c}(x_L)$ such that if $c \in (\underline{c}(x_L), \overline{c}(x_L))$, x_L is an equilibrium outcome.*

These conditions ensure the stability of the division of citizens into "extreme" sets of attendees separated by a set of non-attendees: The cost should be sufficiently small to offer to relatively extreme citizens enough incentives to attend the meeting but it must be high enough to discourage citizens with ideal points relatively close to the equilibrium policy outcome from attending. In that case, the assembly's most preferred policy can be implemented in equilibrium. Recall that in this case the assembly most preferred policy is close to society's most preferred policy.

5. Revisiting Porto Alegre

In the light of the results obtained so far, we next analyze the political events witnessed in Porto Alegre since the Participatory Budgeting experience started. Our claim is that the two types of equilibria characterized in the previous sections represent two different scenarios observed in the city.

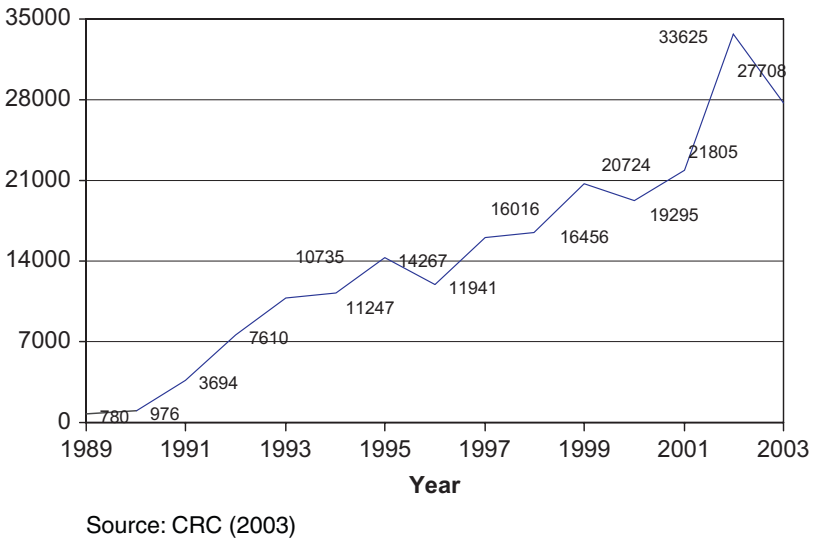

Fig. 2. Total number of participants in the OP, 1989–2003.

At the early stages of the implementation of the Participatory Budgeting system, participation was very low (see Fig. 2) and the priorities selected at the assemblies referred to issues of interest to the working classes: All the investment budget was devoted to cover basic needs (CRC, 2003). Table 1 displays the three top priorities selected in the period 1992–2005. It shows that three areas received the main interest of the population: Paving (that also includes public illumination and garbage collection), basic sanitation (water supply and sewage disposal) and land use regulation (house production, relocation of families living in slums). These priorities simply permuted their rankings in the priority list during the first half of that period.

In this case, the best interpretation of our unidimensional policy space is the population's wealth distribution, because it can be thought as a representation of the citizens' preferences over basic needs. When policy preferences are given by the wealth distribution, the legislator is expected to be extreme with respect to the population, since elected representatives normally belong to high-income segments, especially in Brazil where barriers to entry into politics are important. Our prediction in this case is given by the maximal compromise equilibrium: Few citizens with preferences opposed to the legislator's attend the meeting and they force her to implement a policy close to their most preferred one.

Let us strengthen this interpretation of the policy space by stressing one of the most remarkable features of the Participatory Budgeting experience: The massive participation of those segments of the population typically disengaged from the institutions of representative democracy. In 1998 citizens with a household income not greater than a third of the city average (which was of 13 minimum wages)[19] represented 57% of the participants in the meetings, whereas they only accounted for 32% of the population of Porto Alegre as a whole. This gap broadens if attention is restricted to those citizens with household incomes no greater than two minimum wages, who represent 31% of participants but account for only 12% of the population, as shown in Fig. 3.

As a result of this pattern of participation, the more pressing needs were effectively alleviated: In 1989 in Porto Alegre only 49% of the population was covered by basic sanitation; this percentage rose to 85% in 1996. Over the same period, the proportion of households with water supply reached the 99.5% and the number of students enrolled in elementary schools increased by 190%; five times more housing units were constructed in the period 1993–1996 than in the period 1986–1988.

At later stages of the implementation of the Participatory Budgeting system, once basic needs had been covered, the priorities selected shifted to issues that were also attractive to middle class citizens, like education, health and social services. Thus, it is reasonable to think that the legislator's policy preferences became more aligned with those of the assembly. In fact, over time as participation in the meetings increased (see Fig. 2) the characteristics of the participants changed: The percentage of participants in assemblies with up to four minimum wages fell from 62% in 1995 to 54% in 2000. Thus, it seems reasonable to think that the assembly's preferences become a more representative sample of the population's. In our model, this can be thought of as a change of the relevant policy space, and therefore, as a change in the relevant distribution of the citizens' preferences. Thus, it is also reasonable to think that the preferences of the legislator became more aligned with those of the population. Our prediction in that case is given by the interior equilibrium described in Proposition 5 when the policy preferences of the legislator and those of the society are aligned. Participation in this case is expected to be higher: In the interior equilibrium, at least two citizens or community associations attend the meeting, whereas in the maximum compromise equilibrium that can describe the previous scenario only one individual or association attends. Furthermore, the policy implemented should be closer to the most preferred policy by the assembly

[19] See Marquetti (2000) and CIDADE (2002).

Table 1
Top priorities chosn in Porto Alegre, 1992–2005

Year	1st priority	2nd priority	3rd priority
2005	Housing	Education	Social services
2004	Housing	Social services	Education
2003	Housing	Education	Paving
2002	Housing	Education	Paving
2001	Paving	Housing	Basic sanitation
2000	Housing policy	Paving	Health
1999	Basic sanitation	Paving	Housing policy
1998	Paving	Housing policy	Basic sanitation
1997	Housing policy	Paving	Basic sanitation
1996	Paving	Basic sanitation	Land use regulation
1995	Paving	Land use regulation	Basic sanitation
1994	Land use regulation	Paving	Basic sanitation
1993	Basic sanitation	Paving	Land use regulation
1992	Basic sanitation	Education	Paving

Source: Prefeitura Municipal de Porto Alegre (2005).

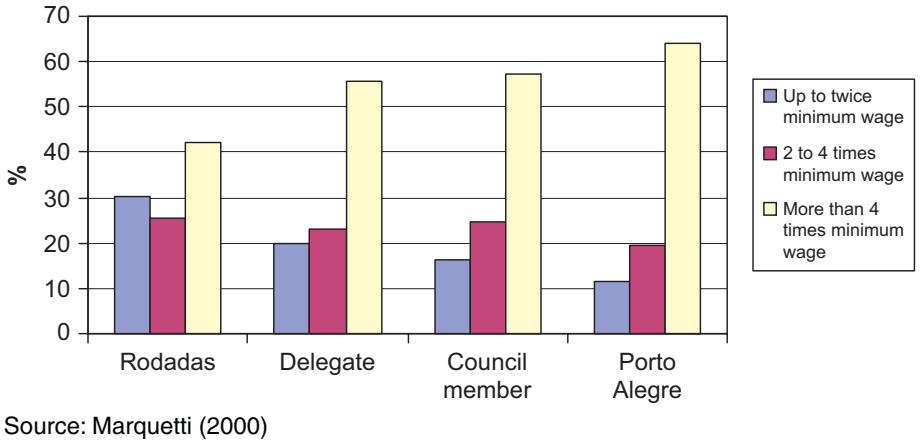

Fig. 3. Income profile of participants in the OP (1998).

compared to the maximal compromise equilibrium. The political scenario in Porto Alegre seems to confirm this, as suggested by the surveys on the citizens' level of satisfaction with the process: In 1995, 60.3% of the citizens believed that the people who participated in the process either "always" or "most of the time" were decisive on public policies; in 2002 this percentage rose to the 69% (CIDADE, 2002).

Furthermore, casual evidence also shows a pattern of confrontation within assemblies that resembles the opposition of blocks of extremists that characterize the interior equilibrium of our model. Santos (1998) reports, that once the lack of physical infrastructures was alleviated, there were increasing levels of conflict of interests (regarding culture and leisure issues) between the participants at the early (regional) meetings, typically coming from low income groups, and those who attend the later (thematic) meetings, more educated and wealthy.

6. Concluding remarks

We have proposed a model of participatory democracy inspired by the system of Participatory Budgeting implemented in the city of Porto Alegre. This experiment, now extended to many other cities worldwide, shows that a participatory system at the local level is indeed possible and can successfully, but not without problems, help govern large communities.

We have formally analyzed the process of participatory democracy by introducing a legislator, with the role of policy implementation, in a formal model of direct democracy. We have shown that this political system is characterized by a relative autonomy between citizens and the legislator: The former group will not be able to implement any policy, and the latter has to acquire calculated compromises. We have also shown how the political events in Porto Alegre can be explained by the two types of different equilibria that can arise in our model.

In fact, a parallel analysis could be used to describe the effect of participatory democracy in countries with different democratic traditions. If we think that the preferences of the society and those of the legislator would be more aligned in countries with mature democracies and less so in countries with weak democratic traditions, our model predicts that

interior equilibrium conditions are more likely to hold in mature democracies while in countries with weak democratic traditions the only equilibrium that may exist is the one with maximal compromise.[20]

We conclude that the main advantage of the system of participatory democracy over direct democracy is that it includes a plausible policy implementation mechanism that the latter lacks. With respect to representative democracy, the main advantage of participatory democracy is that it constitutes a reliable channel for the transmission of information from the citizens to the legislator about their preferences. This is especially relevant when the legislator does not know the precise location of the median voter's ideal point (an assumption repeatedly used in the literature). Hence, the social welfare costs derived from making the "wrong" policy choice will tend to be much smaller under participatory democracy that under representative democracy.

One natural question on robustness refers to our assumption on the legislator's probability of reelection. On the one hand, the fact that the set of citizens who have to decide whether to reelect the legislator or not (the whole population) is different from the set of citizens who attend the meeting may cast some doubts on whether the behavior implied by our reduced form of retrospective voting is consistent with the rest of the features of the model. First of all notice that the legislator is reelected if the median voter decides to reelected her and in most of the cases analyzed this is so. Observe that in any interior equilibrium the policy implemented is very close to the median voter's ideal point, and furthermore, the citizens who decide not to attend the meeting are precisely those who mostly agree with the policy outcome, therefore, it is optimal for them to reelect the legislator. Thus their behavior in this case is consistent with our assumptions. A similar argument applies for any maximal compromise equilibrium with aligned preferences between the legislator and the society, since attendees and non-attendees agree with the policy outcome. Finally, in the maximal compromise equilibrium with non-aligned preferences, the median voter preferences may not be necessarily aligned with those of the attendees.

It should be noticed that the two kinds of equilibria obtained are not an artifact of our specification. For instance, suppose that $P(x_A, x_L) = 1 - (x_L - x_A)^2$ represents the probability with which the legislator is reelected as a function of the assembly's proposal and of the implemented policy. It is easy to show that the optimal decision of the legislator given a policy proposal x_A is $x = (1-\alpha)x_A + \alpha\theta_L$. And given the optimal choice of the delegates we have that the policy implemented in equilibrium will be always in a neighborhood of the legislator's ideal point: $[\alpha\theta_L, (1+\alpha)\theta_L]$. In particular, for $\theta_A < \theta_L$ in equilibrium we will have $x_L = \theta_A$ whenever $\theta_A \geqslant \alpha\theta_L$; and $x_L = \alpha\theta_A$ whenever $\theta_A < \alpha\theta_L$. Similarly, for $\theta_A > \theta_L$ in equilibrium we will have $x_L = \theta_A$ whenever $\theta_A \leqslant (1+\alpha)\theta_L$; and $x_L = (1+\alpha)\theta_A$ whenever $\theta_A > (1+\alpha)\theta_L$. Thus, the results obtained in this case replicate the ones predicted by our model. This shows that the discontinuity obtained in our characterization of equilibria is not implied by the specific probability of reelection we employ but rather by the non-alignment of preferences between legislator and society.

Another question refers to the plausibility of having a legislator whose preferences are not aligned with society's. It may seem natural to expect that only the legislators with desirable preferences will be selected in the long run, so alignment will be ultimately achieved.[21] This would probably be the case under perfect competition in the political arena. But in the market of political candidates, barriers to entry abound. It is very rare for representatives to come from average or below-average income levels. Entering political competition entails substantial costs, mostly financial. If preferences over policies are given by wealth, then it is not reasonable to expect preferences within the pool of potential candidates to be similar to those of the society as a whole. Non-alignment of preferences between the legislator and the society induces instability, and it could imply a change of candidate or even the abandonment of the system of participatory democracy.

The reader may find the main institutional features of participatory democracy striking at first glance: Only the opinions of those who participate at the meetings are taken into account. On the contrary, in a representative democracy everybody's opinion can be heard through the simple act of voting (surely, casting a vote is cheaper than attending an assembly). This comparison is misleading for two reasons. First, as mentioned before, many of the participants in the assemblies are representing families or communities associations so it is not correct to think that only a few thousand opinions are heard. Another reason is that, as the low level of turnout in Western democracies indicates, this alleged advantage of representative democracy is doubtful: It is well known that the fraction of population systematically disenfranchised from the representative processes are the less educated and wealthy, precisely those segments overwhelmingly represented at the popular assemblies in Porto Alegre.

Acknowledgments

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[20] We thank an anonymous referee for bringing up this point.

[21] This would be the case if the legislators where selected according to the predictions of the citizen-candidate models by Osborne and Slivinski (1996) and Besley and Coate (1997).

Appendix A

Proof of Proposition 1: Equilibrium policies. In order to find the policies implemented by the legislator in equilibrium we combine the optimal choice functions of the legislator and the assembly:

$$x_L(x_A) = \begin{cases} x_A + B & \text{if } B \leqslant |x_A - \theta_L| \leqslant b + B \text{ and } x_A \leqslant \theta_L, \\ x_A - B & \text{if } B \leqslant |x_A - \theta_L| \leqslant b + B \text{ and } x_A \geqslant \theta_L, \\ \theta_L & \text{otherwise,} \end{cases}$$

$$x_A(\theta_A) = \begin{cases} \theta_L - b - B & \text{if } \theta_A \leqslant \theta_L - b, \\ \theta_A - B & \text{if } \theta_L - b \leqslant \theta_A \leqslant \theta_L, \\ \theta_A + B & \text{if } \theta_L \leqslant \theta_A \leqslant \theta_L + b, \\ \theta_L + b + B & \text{if } \theta_L + b \leqslant \theta_A. \end{cases}$$

We have to consider four different cases:

(i) if $\theta_A \leqslant \theta_L - b$, then the best response of the assembly is to propose $x_A = \theta_L - b - B$. In this case, $\theta_L - x_A = b + B$ and $x_A \leqslant \theta_L$, thus the legislator implements $x_L = x_A + B = \theta_L - b$;
(ii) if $\theta_L - b \leqslant \theta_A \leqslant \theta_L$, then the best response of the assembly is to propose $x_A = \theta_A - B$. In this case, $B = \theta_A - x_A \leqslant \theta_L - x_A = \theta_L - \theta_A + B \leqslant b + B$ and $x_A \leqslant \theta_L$, thus the legislator implements $x_L = x_A + B = \theta_A$;
(iii) if $\theta_L \leqslant \theta_A \leqslant \theta_L + b$, then the best response of the assembly is to propose $x_A = \theta_A + B$. In this case, $B = x_A - \theta_A \leqslant x_A - \theta_L = \theta_A + B - \theta_L \leqslant b + B$ and $x_A \geqslant \theta_L$, thus the legislator implements $x_L = x_A - B = \theta_A$;
(iv) if $\theta_L + b \leqslant \theta_A$, then the best response of the assembly is to propose $x_A = \theta_L + b + B$. In this case, $\theta_L - x_A = b + B$ and $x_A \geqslant \theta_L$, thus the legislator implements $x_L = x_A - B = \theta_L + b$.

Therefore, in equilibrium the legislator will implement the most preferred policy of the assembly only when it belongs to the interval $[\theta_L - b, \theta_L + b]$, otherwise the only policies that will be implemented in equilibrium are $\theta_L - b, \theta_L + b$. □

Proof of Proposition 2: Non-Attendance. First we show that when $c > b$ there is an equilibrium in which nobody attends the meeting.

Suppose that nobody attends the meeting. In this case the legislator will implement her ideal point, thus $x_L = \theta_L$. Given the result in Proposition 1, any citizen contemplating the option of attending will move the final policy by b at most. If the ideal point of this citizen were within the interval $[\theta_L - b, \theta_L + b]$ he would be better off by not attending since it costs c to do so. On the other hand, consider a citizen within the interval $[\theta_L - b, \theta_L + b]$. If he were to attend, he would be the only participant and he would be able to obtain his ideal point θ_i. Thus he would move the policy output a distance $|\theta_L - \theta_i| < b < c$ towards him, implying again that he is better off by not attending.

Next we will show that nobody attending the meeting is the unique equilibrium of the game when $c > b$.

Let $x_L(X) = x_L(\theta_A(X))$ denote the policy implemented as a function of the set of citizens who attend the meeting, X, given that citizens and legislator are playing their best responses at the last two stages of the game.

Observe that for any set of attendees in equilibrium X, and for any $\theta_i \in X$ we must have that $V_i(x_L(X), 1) = -|x_L(X) - \theta_i| - c > V_i(x_L(X - \{\theta_i\}), 0) = -|x_L(X - \{\theta_i\}) - \theta_i|$, that is $|x_L(X) - x_L(X - \{\theta_i\})| > c$. Since $x_L \in [\theta_L - b, \theta_L + b]$ we have that:

(i) If $\theta_i \leqslant \theta_L$, this condition implies that $\theta_L - b \leqslant x_L(X) \leqslant \theta_L + b - c < \theta_L$.
(ii) If $\theta_i \geqslant \theta_L$, this condition implies that $\theta_L < \theta_L - b + c \leqslant x_L(X) \leqslant \theta_L + b$.

These two conditions cannot hold simultaneously. These implies that in equilibrium we must have that either $X = \{\theta_i : \theta_i \leqslant \theta_L\}$ or $X = \{\theta_i : \theta_i \geqslant \theta_L\}$. Suppose that $X = \{\theta_i : \theta_i \leqslant \theta_L\}$, then since all $\theta_i \geqslant \theta_L$ do not attend the meeting we must have that $x_L(X) \leqslant \theta_L$ and $x_L(X - \{\theta_i\}) \leqslant \theta_L$ for all $\theta_i \in X$. But this implies that $|x_L(X) - x_L(X - \{\theta_i\})| < b < c$ which contradicts the condition that makes attendance optimal. Similarly we can prove that $X = \{\theta_i : \theta_i \geqslant \theta_L\}$ leads to a contradiction. Therefore, if $c > b$ there cannot be an equilibrium in which some citizens attend. □

Proof of Proposition 3: Attendance in equilibrium. We will show that if nobody attends the meeting any citizen with ideal point θ_i such that $|\theta_i - \theta_L| > c$ would be better off attending the meeting. Since we have assumed that there is at least a citizen with $\theta_i = 0$ and at least a citizen with $\theta_i = 1$, we will always have someone that has a profitable deviation. Suppose that $c < b$ and nobody attends the meeting, that is, $X = \emptyset$. Consider a citizen with θ_i such that $b > |\theta_L - \theta_i| > c$. Since he does not attend the meeting his payoff is $V_i(\theta_L, 0) = -|\theta_L - \theta_i|$. If he were to attend, he would be the only participant. Since his

ideal point is within the interval $[\theta_L - b, \theta_L + b]$ he would be able to obtain his ideal point θ_i as the policy output, thus his payoff would be $V_i(\theta_i, 1) = -|\theta_i - \theta_i| - c = -c$. Since $c < |\theta_L - \theta_i|$, we have that $V_i(\theta_L, 0) = -|\theta_L - \theta_i| < V_i(\theta_i, 1) = -c$. Therefore, he would be better off attending.

Now consider a citizen with θ_i such that $\theta_i \leqslant \theta_L - b$. Since he does not attend the meeting his payoff is $V_i(\theta_L, 0) = -|\theta_L - \theta_i| = \theta_i - \theta_L$. If he were to attend, he would be the only participant. Since his ideal point is not within the interval $[\theta_L - b, \theta_L + b]$ the best policy he could obtain is $x_L = \theta_L - b$, thus his payoff would be $V_i(\theta_L - b, 1) = -|\theta_L - b - \theta_i| - c = \theta_i - \theta_L + b - c$. Since $c < b$ we have that $V_i(\theta_L, 0) < V_i(\theta_L - b, 1)$. Therefore, he would be better off attending.

Similarly we can prove that any citizen with $\theta_i \geqslant \theta_L + b$ would be better off attending the meeting. Therefore, nobody attending the meeting cannot be an equilibrium. □

Proof of Proposition 4: Symmetric equilibrium. We are assuming that $F(\theta_i)$ is symmetric around $\theta_L = \frac{1}{2}$ and $c < b$. We will show that there is a symmetric equilibrium in which $X = \{\theta_i : |\theta_i - \theta_L| > c\}$, by proving that all $\theta_i \in X$ are better off attending and all $\theta_i \notin X$ are better off not attending.

Suppose that $X = \{\theta_i : |\theta_i - \theta_L| > c\}$ and consider a citizen $\theta_i \in X$ such that $\theta_i < \frac{1}{2} - c$. Since he attends the meeting his payoff is $V_i(\frac{1}{2}, 1) = -|\theta_i - \frac{1}{2}| - c = \theta_i - \frac{1}{2} - c$. If he would not attend his utility would be $V_i(x_L(X - \{\theta_i\}), 0) = -|\theta_i - x_L(X - \{\theta_i\})| = \theta_i - x_L(X - \{\theta_i\})$. Thus he is better off attending if and only if $x_L(X - \{\theta_i\}) > \frac{1}{2} + c$. We will show that this is always the case.

Since $x_L(X - \{\theta_i\}) = \min\{\theta_A(X - \{\theta_i\}), \frac{1}{2} + b\}$, and we have that $\theta_A(X - \{\theta_i\}) = \mathit{median}(X - \{\theta_i\}) > \frac{1}{2} + c$ and $b > c$, then we must have that $x_L(X - \{\theta_i\}) > \frac{1}{2} + c$.

Now consider a citizen $\theta_i \notin X$ such that $\frac{1}{2} - c < \theta_i < \frac{1}{2}$. Since he does not attend his utility is $V_i(\frac{1}{2}, 0) = -|\theta_i - \frac{1}{2}| = \theta_i - \frac{1}{2}$. If he was to attend he would obtain $V_i(x_L(X \cup \{\theta_i\}), 1) = -|\theta_i - x_L(X \cup \{\theta_i\})| - c = -c$ since in this case he would become the median of the assembly, that is, $\theta_A(X \cup \{\theta_i\}) = \mathit{median}(X \cup \{\theta_i\}) = \theta_i$ and since $\theta_i \in [\theta_L - b, \theta_L + b]$ his ideal point would be implemented $x_L(X \cup \{\theta_i\}) = \theta_i$. Thus, he is better off not attending because $\frac{1}{2} - c < \theta_i$. □

Proof of Proposition 5: Maximal compromise equilibrium. We will only prove part (a). The proof of part (b) is almost identical and is left to the reader.

First we show that if $x_L(X) = \theta_L + b$ is the equilibrium outcome, then X must be a singleton. Suppose that the equilibrium outcome is $x_L(X) = \theta_L + b$. Then for all $\theta_i \in X$ such that $\theta_i < \theta_L - b$ we have that $x_L(X - \{\theta_i\}) = x_L(X)$, therefore we must have that $V_i(x_L(X - \{\theta_i\}), 0) > V_i(x_L(X), 1)$. Therefore, they would be better off not attending and in equilibrium we must have that $X \subseteq \{\theta_i : \theta_i \geqslant \theta_L + b\}$.

Given that, for all $\theta_i \in X$ such that $\theta_i \geqslant \theta_L + b$ we have that

$$x_L(X - \{\theta_i\}) = \begin{cases} x_L(X) & \text{if } |X| > 1, \\ \theta_L & \text{if } |X| = 1. \end{cases}$$

This implies that $V_i(x_L(X - \{\theta_i\}), 0) > V_i(x_L(X), 1)$ as long as $|X| > 1$. Therefore, they would be better off not attending as long as $|X| > 1$. Therefore, the only possibility for equilibrium is $|X| = 1$.

Next we show that there is an equilibrium with $x_L(X) = \theta_L + b$ iff $\theta_L < \frac{1}{2} - b + c$. We already know that if $x_L(X) = \theta_L + b$ is the equilibrium outcome, we must have $X = \{\theta_i\}$ with $\theta_i \geqslant \theta_L + b$. Observe that if a citizen θ_i with $\theta_i \geqslant \theta_L + b$ is the only citizen who attends the meeting his best proposal is $x_A = \theta_L + b + B$, the policy implemented is $x_L = \theta_L + b$ and he obtains $V_i(\theta_L + b, 1) = -|\theta_i - \theta_L - b| - c = \theta_L + b - \theta_i - c$. If he was not to attend he would obtain $V_i(\theta_L, 0) = -|\theta_i - \theta_L| = \theta_L - \theta_i$. Thus he is better off attending since $b > c$.

Suppose that $X = \{\theta_i\}$ with $\theta_i \geqslant \theta_L + b$. Observe that for any $\theta_j \geqslant \theta_L + b, \theta_j \neq \theta_i$ we have that $x_L(X \cup \{\theta_j\}) = \theta_L + b$, thus they would gain nothing by attending, and they would have to pay the cost. Therefore, they are all better off not attending.

Now consider a θ_j with $\theta_j < \theta_L + b$. If we show that $\theta_j = 0$ prefers not to attend the meeting, we will have that all $\theta_j < \theta_L + b$ prefer not to attend.

If $\theta_j = 0$, if he does not attend the meeting he obtains $V_0(x_L(X), 0) = -|\theta_L + b| = -\theta_L - b$; if he were to attend the policy outcome would be $x_L(X \cup \{0\}) = \max\{\theta_i/2, \theta_L - b\}$ and thus he would obtain

$$V_0(x_L(X \cup \{0\}), 1) = -\left|\max\left\{\frac{\theta_i}{2}, \theta_L - b\right\}\right| - c = -\max\left\{\frac{\theta_i}{2}, \theta_L - b\right\} - c.$$

We have that he is better off not attending iff $-\theta_L - b > -\max\{\theta_i/2, \theta_L - b\} - c$ iff $\theta_L + b - c < \max\{\theta_i/2, \theta_L - b\}$.

Notice that if $\max\{\theta_i/2, \theta_L - b\} = \theta_L - b$ the inequality is never satisfied. Thus it is necessary and sufficient that we have $\theta_i/2 > \theta_L + b - c$, that is $\theta_i > 2(\theta_L + b - c)$. And there would be such a citizen if and only if $2(\theta_L + b - c) < 1$ iff $\theta_L < \frac{1}{2} - b + c$.

Thus we have shown that there is an equilibrium with $x_L(X) = \theta_L + b$ iff $\theta_L < \frac{1}{2} - b + c$. And in this equilibrium we have that $X = \{\theta_i\}$ with $\theta_i > \max\{\theta_L + b, 2(\theta_L + b - c)\}$.

Similarly we could show that there is an equilibrium with $x_L(X) = \theta_L - b$ iff $\theta_L > \frac{1}{2} + b - c$. And in this equilibrium we would have that $X = \{\theta_i\}$ with $\theta_i < \min\{\theta_L - b, 2(\theta_L - b + c) - 1\}$. □

Proof of Proposition 6: Non-participation of the represented. Let us first state the following auxiliary lemma:

Lemma 1. *Suppose that $c < b$ and $x_L \in (\theta_L - b, \theta_L + b)$ is the equilibrium outcome. Then for any citizen i with ideal point θ_i who does not attend, the number of attendees j with $\theta_j > \theta_i$ and the number of attendees k with $\theta_k < \theta_i$ are equal.*

Proof of Lemma 1. This lemma follows directly from the next two steps with their symmetric counterparts.

Step 1: For any distribution of citizens' ideal points, if $c < b$, the equilibrium outcome satisfies $x_L(X) \in (\theta_L - b, \theta_L + b)$, and there is a $\theta_i < x_L(X)$ (or a $\theta_i > x_L(X)$) that is not attending, then he must leave either the same number of attendees at each side or one more attendee to her right (left).

Suppose that $\theta_i < x_L(X)$ is not attending, and there are k attendees on her left and $k + l$ attendees on her right with $l > 1$. Let $\theta_1, \ldots, \theta_l$ denote the first l attendees on the right of θ_i, then $x_L(X) = median(\theta_1, \ldots, \theta_l)$.

Suppose that l is odd. Then we have that $x_L(X) = \theta_{(l+1)/2}$ and $x_L(X \cup \theta_i) = x_L(X - \theta_{(l+1)/2+1}) = \max\{\theta_{(l+1)/2-1} + \theta_{(l+1)/2}/2, \theta_L - b\}$. But we need $x_L(X) - x_L(X - \theta_{(l+1)/2+1}) > c$ because $\theta_{(l+1)/2+1}$ is attending, and $x_L(X) - x_L(X \cup \theta_i) < c$ because θ_i is not attending. Therefore we have a contradiction.

Now suppose that l is even. Then we have that $x_L(X) = \theta_{l/2} + \theta_{l/2+1}/2$ and $x_L(X \cup \theta_i) = x_L(X - \theta_{l/2+1}) = \max\{\theta_{l/2}, \theta_L - b\}$. But we need $x_L(X) - x_L(X - \theta_{l/2+1}) > c$ because $\theta_{l/2+1}$ is attending, and $x_L(X) - x_L(X \cup \theta_i) < c$ because θ_i is not attending. Therefore we have a contradiction. Thus, we have proved that in equilibrium we must have $l \leqslant 1$.

Step 2: For any distribution of citizens' ideal points, if $c < b$, the equilibrium outcome satisfies $x_L(X) \in (\theta_L - b, \theta_L + b)$, and there is a $\theta_i < x_L(X)$ that is not attending, then all θ_j such that $\theta_i < \theta_j \leqslant x_L(X)$ are not attending.

Suppose that $\theta_i < x_L(X)$ is not attending and there is a θ_j with $\theta_i \leqslant \theta_j < x_L(X)$ that is attending. By the previous lemma θ_i must leave either k attendees on both sides or k attendees to his left and $k + 1$ attendees to his right.

In the first case, it implies that θ_j must leave k attendees to his left and $k - 1$ attendees to his right. Which would imply that $x_L(X) = median(X) < \theta_j$. A contradiction since $\theta_j < x_L(X)$.

In the second case, it implies that θ_j must leave k attendees to his left and k attendees to his right. Which would imply that $x_L(X) = median(X) = \theta_j$. A contradiction since $\theta_j < x_L(X)$.

Now we will show that if any $\theta_i < x_L(X) - c$ does not attend the meeting, the equilibrium conditions are not satisfied. A similar reasoning can be used to show the symmetric counterpart for any $\theta_i > x_L(X) - c$.

Suppose that there is a $\theta_i \notin X$ such that $\theta_L - b < \theta_i < x_L(X) - c$. His utility is $V_i(x_L(X), 0) = -|\theta_i - x_L(X)| = \theta_i - x_L(X) < -c$ Since he is not attending the meeting, by Lemma 1 there must be half of the attendees' ideal points to his right and half to his left θ_i If he was to attend he would become the median of the attendees and $x_L(X \cup \{\theta_i\}) = \theta_i$, thus his utility would be $V_i(x_L(X \cup \{\theta_i\}), 1) = -|\theta_i - x_L(X \cup \{\theta_i\})| - c = -c$. Therefore, he would be better off attending, and this is a contradiction.

Finally, suppose that there is a $\theta_i \notin X$ such that $\theta_i < \theta_L - b < x_L(X) - c$. His utility is $V_i(x_L(X), 0) = -|\theta_i - x_L(X)| = \theta_i - x_L(X)$ and since he is not attending the meeting, by Lemma 1 he must also have that half of the attendees' ideal points to his right and half to his left. Thus, if he was to attend he would become the median of the attendees and$x_L(X \cup \{\theta_i\}) = \theta_L - b$. Thus his utility would be $V_i(x_L(X \cup \{\theta_i\}), 1) = -|\theta_i - x_L(X \cup \{\theta_i\})| - c = \theta_i - (\theta_L - b) - c > \theta_i - (x_L(X) - c) - c = \theta_i - x_L(X)$. Therefore, he would be better off attending, and this is a contradiction. □

Corollary 2 (*Interior equilibrium policies*). *Suppose that $x_L(X) \in (\theta_L - b, \theta_L + b)$. From Lemma* 1 *we know that we must have $x_L X) = \theta_l + \theta_r/2$ for some θ_l and θ_r that are attending the meeting. Since θ_l and θ_r are attending the meeting, by Proposition* 6 *we must also have that $\theta_l, \theta_r \notin (x_L(X) - c, x_L(X) + c)$.*

Since θ_r is attending his utility is $V_r(x_L(X), 1) = -|\theta_r - x_L(X)| - c = x_L(X) - \theta_r - c$. If θ_r was not attending, θ_l would be the median of the attendees, thus $\theta_A(X - \{\theta_r\}) = \theta_l$. Since $\theta_l \leqslant x_L(X) - c < \theta_L - b$ we have that the policy that legislator would implement in this case is $x_L(X - \{\theta_r\}) = \theta_L - b$. Thus his utility in this case would be $V_r(x_L(X - \{\theta_r\}), 0) = -|\theta_r - \theta_L + b| = \theta_L - b - \theta_r$. Since we assumed that $x_L(X) - c < \theta_L - b$, we have that θ_r would be better off not attending the meeting, which is a contradiction. Similarly we can prove that in equilibrium we must have $x_L(X) + c < \theta_L + b$.

Corollary 3 (*Alignment is needed*). *By Proposition* 6 *we know that in equilibrium all citizens with ideal points to the left of $x_L(X) - c$ will attend, and by Corollary* 2 *we know that $\theta_L - b < x_L(X) - c$. This implies that when $\theta_m < \theta_L - b$ we have more than half of the population to the left of $x_L(X) - c$,and they are all attending the meeting. Combining Proposition* 6 *and Corollary* 2 *there should be exactly half of the attendees to the left of $x_L(X) - c$. This is a contradiction.*

Proof of Proposition 7: Interior equilibrium. Suppose that $x_L \in S$, that is, $x_L = \theta_r + \theta_l/2$ for some θ_l and θ_r such that $|\{\theta_i : \theta_i \leqslant \theta_l\}| = |\{\theta_i : \theta_i \geqslant \theta_r\}|$. Observe that if θ_l and θ_r attend the meeting their payoffs are $V_R(x_L(X), 1) = V_L(x_L(X), 1) = \theta_l - \theta_r/2 - c$. If one of them decides not to attend his payoff is $V_i(x_L(X - \{\theta_i\}), 0) = \theta_l - \theta_r$ for $i = L, R$. Thus, in order to have both citizens attending in equilibrium we must have $\theta_l - \theta_r/2 - c > \theta_l - \theta_r$. Hence we need c to satisfy the following

condition $c<\overline{c}=(\theta_r-\theta_l)/2$. Observe that $\overline{c}<b$. Ifθ_l and θ_r are attending, then all the other citizens such that either $\theta_i\leqslant\theta_l$ or $\theta_i\geqslant\theta_r$ are better off attending.

Next we will show that all citizens such that $\theta_l<\theta_i<\theta_r$ are better off not attending the meeting: Let θ_{l+1} denote the ideal point of the voter next to θ_l on his right and let θ_{r-1} denote the ideal point of the voter next to θ_r on his left. If they are not attending the meeting their payoffs are $V_{L+1}(x_L(X),0)=-|\theta_{l+1}-\theta_l+\theta_r/2|$ and $V_{R-1}(x_L(X),0)=-|\theta_{r-1}-\theta_l+\theta_r/2|$. If one of them decides to attend his payoffs is $V_i(x_L(X\cup\{\theta_i\}),1)=-c$ for $i=l+1,r-1$. Thus, in order to have both not attending in equilibrium we must have $|\theta_{l+1}-\theta_l+\theta_r/2|<c$ and $|\theta_{r-1}-\theta_l+\theta_r/2|<c$. Hence we need c to satisfy the following condition $c>\underline{c}=\max\{|\theta_{l+1}-\theta_l+\theta_r/2|,|\theta_{r-1}-\theta_l+\theta_r/2|\}$. Observe that $0\leqslant\underline{c}<\overline{c}$.If θ_{l+1} and θ_{r-1} are better off not attending, then so are all the other citizens such that either $\theta_i\in[\theta_{l+1},\theta_{r-1}]$. Thus, if $c\in(\underline{c},\overline{c})$ there is an equilibrium with $X=\{\theta_i:\theta_i\leqslant\theta_l\}\cup\{\theta_i:\theta_i\geqslant\theta_r\}$ and $x_L(X)=(\theta_r+\theta_l)/2$. □

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