# An Automated Model of Government Formation*

Enriqueta Aragonès†
IAE-CSIC

Pilar Dellunde‡
UAB and IIIA-CSIC

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**Abstract**

In this paper we propose a formal model of bargaining for government formation in a parliamentary democracy that permits the analysis of the effects of a large class of bargaining strategies on the possibility of reaching agreements and on the policy compromise of the members of the government coalition that forms. We also propose a complementary algorithm that, applied to the proposed model, would allow to implement the simulations of the interplay of different sets of strategies. The implementation of the combination described above should shed some light on the performance of the different strategies according to the benefit they produce for the parties.

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†Institut d'Anàlisi Econòmica, CSIC. Campus UAB, 08193 Bellaterra (Spain). enriqueta.aragones@iae.csic.es

‡Institut d'Investigació en Intel·ligència Artificial, CSIC. Campus UAB, 08193 Bellaterra (Spain). pilar.dellunde@uab.cat

*"There can be no doubt at all that the government formation process, which begins with a particular election result that leaves open many coalition possibilities and ends with the formation of a government comprising a particular combination of parties, is one of the fundamental processes of European parliamentary democracy. Understanding how a given election result leads to a given government is when all is said and done, simply one of the most important substantive projects in political science."*

*Laver and Schofield (1990,89)*

# 1 Introduction

The aim of this paper is to propose a suitable procedure to analyze the process of government formation in a parliamentary democracy. In particular, we would like to characterize a large class of bargaining strategies suitable for the agents involved in this process and analyze their performance.

The process of government formation in parliamentary democracies is characterized by many institutional features that lie in the definition of the process itself: the choice of the formateur (the party in charge of initiating the negotiations), the support needed to form a government (the proportion of parliamentary votes required for the investiture of the executive), the existence of a limited amount of time for the negotiation after which either a new formateur might be selected, or a new election is called, among others.

There are other institutional features that indirectly may affect this process such as: the support needed for policy implementation (the proportion of parliamentary votes required to pass a law), the power of the executive in terms of policy implementation (relative to the power of the legislative), the existence of different governmental levels, the requirements to call a vote of confidence, the possibility of calling early elections,... The centrality of the process of government formation is such that almost all institutional features might be thought of relevant for it.

Other features might have an effect on the result of the parties' negotiation for government formation, such as the current characteristics of the party system. The number of parties that have parliamentary representation as well as their ideologies and the distribution of parliament seats among them are specific features of great relevance to determine the process of government formation.

The number and type of issues that appear as relevant during the bargaining process might also have an important effect on the negotiation process and its outcome. We might distinguish between quantitative issues, that refer to the distribution of the value of holding office (more specifically, they might refer to the distribution of executive positions among party members, for instance, the government presidency, the parliament presidency, the cabinet ministers,...) and qualitative issues, that refer to issues that involve an ideological dimension (for instance, the amount of public service to be provided, the level of taxation,...)

In addition there are intangible issues such as: getting the deal done, making voters happy, standing by one's principles, being fair, beating the competitors, looking good to the constituency, preserving one's reputation, setting a precedent,... These issues might also play a central role during the process of government formation, however they will be treated in a different way since they mostly affect the way in which parties make decisions.

Finally, the specific way in which the negotiations take place might also have an important effect on the process and its outcome. In particular, the negotiation can be performed sequentially, given an order of the issues, simultaneously by subsets of issues,... And the different possible coalitions might also bargain sequentially or simultaneously.

Hence, when studying the process of government formation in parliamentary democracies there are a lot of features that have to be taken into account. The existing theories of bargaining in economics, including Rubinstein's (1982) seminal paper and all its extensions[1], offer a lot of insights that have been used by the emerging literatures on legislative bargaining and government formation.

The legislative bargaining literature includes Baron and Ferejohn's (1989) basic model of legislative bargaining; Baron's (1991a) application to pork barrel; Romer and Rosenthal (1978 and 1979) assume an exogenous status quo; Eraslan (2002) shows the uniqueness of payoffs for stationary equilibria; Banks and Duggan (2000 and 2002) extend it to bargaining over policy; Calvert (1989) analyzes legislative reciprocity, and Jackson and Moselle (2002) combine bargaining over policy with distributive bargaining.

The literature on government formation includes Baron and Ferejohn (1991b) which extends their previous work to explain government formation; Baron (1993) assumes endogenous parties; Diermeier and Myerson (1994) introduce a veto player; Laver and Schofield (1990) and Laver and Shepsle (1996) analyze the role of the formateur; Austen-Smith and Banks (1988) introduce strategic voting; Merlo (1997) studies the effects of deadlines and delays; Diermeier and Merlo (2000) consider dynamic features; Baron and Diermeier (2001) analyze a multidimensional space; Diermeier, Eraslan and Merlo (2003) study the effects of the investiture vote and the no-confidence vote; Aragones (2007a and 2007b) analyzes a two dimensional model and applies to a real world case (catalan government).

Most of these works focus on the effect of particular features of the process of government formation. We provide a procedure that allows to consider a large numbers of features and to compare their effects. For this purpose, we describe a formal model of government formation and combine it with an automated and tractable negotiation mechanism for autonomous agents. Our multilateral bar-

[1] Extensions of Rubinstein model include:
Sutton (1986) and Osborne and Rubinstein (1990) that discuss the effects of assuming unanimity; Krishna and Serrano (1996) that generalize it to any number of players; Jun (1987) and Chae and Young (1988 and 1994) introduce bilateral negotiations, and Chaterjee, Dutta, Ray, Sengupta (1993) introduce transferable utility.

gaining model is based on the bilateral automated negotiation by Faratin, Sierra and Jennings (1998) and extends it to a multi-agent automated negotiation.

The rest of the paper is organized as follows: the next section presents the formal model. Section 3 describes the negotiation process. Section 4 describes the automated negotiation mechanism. Section 5 we introduce the basic hypotesis to be tested in future experimental research. Finally section 6 offers some concluding remarks.

# 2 The formal model

We consider a finite set of political parties $\mathbf{P} = \{1, 2, ..., P\}$ with parliament representation. We assume that an election has already taken place, and the proportion of parliament seats that party $p$ has obtained is given by $v_p$. We assume that $\sum_{p \in \mathbf{P}} v_p = 1$ and $0 \leq v_p < \frac{1}{2}$ for all $p \in \mathbf{P}$. We do not consider the possibility that a party has a majority of the seats, that is, $v_p > \frac{1}{2}$ for some party $p$, since the analysis of this case would lead to trivial results[2].

We assume that the issues that parties care about are of two kinds: qualitative issues and quantitative issues. Let $\mathbf{Q} = \{1, 2, ..., Q\}$ denote the set of qualitative issues, and let $\mathbf{S} = \{1, 2, ...S\}$ denote the set of quantitative issues. We assume that both $Q$ and $S$ are natural numbers. Let $q$ and $s$ denote a generic element of sets $\mathbf{Q}$ and $\mathbf{S}$ respectively.

We represent an issue $q \in \mathbf{Q}$ by a real interval $[0, \overline{q}] \subset \Re$. Each $x \in [0, \overline{q}]$ represents a specific policy position on issue $q$. We assume that each political party has a most preferred policy over each one of the qualitative issues, derived from the party's ideological principles. Let $q_p \in [0, \overline{q}]$ denote the most preferred policy position by party $p$ on issue $q$. The elements of $\mathbf{Q}$ can be thought of as representing the ideological issues over which political parties have preferences. We assume that the parties' preferences on this space as such that different political parties have different ideal points.

A quantitative issue, $s \in \mathbf{S}$ can be thought of as a set of executive positions that correspond to the same rank. For instance, the government presidency, the parliament presidency, the cabinet ministers,... We represent an issue $s \in \mathbf{S}$ by an interval $[0, \overline{s}] \subset \Re$. Each quantitative issue $s$ represents a specific rank of executive positions and $\overline{s}$ represents the total amount of seat of rank $s$ available. We assume that *ceteris paribus* the payoffs of all political parties increase with the number of executive positions that they obtain and with the rank of the positions that they obtain. An allocation of executive positions among parties will be given by a specific allocation of positions of each rank.

Thus, parties have to bargain over a multidimensional policy space of $Q + S$ dimensions, where each dimension represents a different issue.

We assume that parties are mainly concerned about holding office, that is, they care about being members of the governing coalition and about obtaining

[2] We rule out the possibility that $v_p = \frac{1}{2}$ because it does not represent a real situation since most parliaments have an odd number of seats.

executive positions on the quantitative issues. On the other hand, the policy implemented on the qualitative issues may affect the vote support of parties that are members of the governing coalition in future elections, and therefore their future payoff. Naturally, parties that are not members of the governing coalition are perceived by voters as detached from the implemented policies, because voters do not consider them responsible of the implemented policies. Therefore, it is reasonable to think that their payoffs are not affected by the government's policy choices. It is normally the case that parties that are not members of the governing coalition are not allocated any executive positions. This implies that their payoffs are not affected by the government choice on the quantitative issues either. Thus, the payoffs of parties that are not members of the governing coalition would not be affected by any of the government's choices. We normalize the utility of a party that is not a member of the governing coalition to zero, and we represent the utility that party $p$ obtains if it becomes a member of the governing coalition by $U_p(\mathbf{q},\mathbf{s})$ where $(\mathbf{q},\mathbf{s})$ represents the vector of policies implemented by the governing coalition.

Therefore, the payoff function of party $p$ can be defined as follows:

$$V_p(G,(\mathbf{q},\mathbf{s})) = \begin{cases} 0 & if \quad p \notin G \\ U_p(\mathbf{q},\mathbf{s}) & if \quad p \in G \end{cases}$$

Where $G$ denotes a governing coalition. A governing coalition must be decisive in terms of policy choices, that is, it must be supported by at least a majority of seats of the parliament. Since we assume that no party obtains a majority of the seats, parties are supposed to form coalitions that have the support of at least a majority of the parliament in order to form a decisive coalition. Within the proposed governing coalition, the members will have to negotiate a policy compromise.

$\mathbf{q} = (x_1, ..., x_Q) \in \Re^{|\mathbf{Q}|}$ is a Q-dimensional vector and each component represents a policy compromise on each qualitative or ideological issue. $\mathbf{s} \in \Re^{|\mathbf{S}\times\mathbf{P}|}$ represents the allocation of executive positions among the members of a coalition. $\mathbf{s}$ has $|\mathbf{S}|$ components, one for each quantitative issue[3]. Each component of $\mathbf{s} = (s_1, ..., s_S)$ is represented by a vector of dimension $\mathbf{P}$, $s_j = \left(s_j^1, ...s_j^P\right)$ where $s_j^p \geq 0$ represents the number of executive positions of rank $j$ allocated to party $p$. We order the different ranks in a decreasing manner, that is, we assume that higher ranked positions are represented by lower values of $j$. Therefore, the dimension of a vector of policy proposals $(\mathbf{q},\mathbf{s})$ is $r = |\mathbf{Q}| + |\mathbf{S}\times\mathbf{P}|$.

Only parties that are members of the governing coalition may obtain a positive allocation of executive positions of any rank. The sum of the components of the vector corresponding to issue $s_j$ cannot be larger than $\overline{s_j}$, thus for all $\mathbf{s} \in \mathbf{S}$ and for all $j$ we must have $\sum\limits_{p\in\mathbf{P}} s_j^p = \sum\limits_{p\in G} s_j^p \leq \overline{s_j}$. Let $\mathbf{s}^p$denote the $p$-dimensional

[3] We will assume that the set of positions to be allocated is exogenous, that is, the governing coalition can neither create new positions to be allocated, nor can increase the number of positions at any level, even though this is not what happens in real situations. This will be left for future work

vector whose components represent the allocations of executive positions of each rank to party $p$.

We assume that the utility that party $p$ obtains if it becomes a member of the governing coalition when the implemented policy is $(\mathbf{q}, \mathbf{s})$ is given by

$$U_p(\mathbf{q}, \mathbf{s}) = Q_p(\mathbf{q}) + K_p S_p(\mathbf{s})$$

where

$$Q_p(\mathbf{q}) = -\sum_{q \in \mathbf{Q}} \pi_p(q) \cdot (x_q - q_p)^2$$

represents the payoff that party $p$ derives from the compromise reached on the qualitative issues[4]; $q$ represents the policy compromise on issue $q$ by the governing coalition, and $\pi_p(q)$ represents the weight that party $p$ assigns to issue $q$, such that $\pi_p(q) > 0$ and $\sum_{q \in \mathbf{Q}} \pi_p(q) = 1$.

According to this utility function, parties' preferences over policies are single peaked and convex. The parameters $\pi_p(q)$ represent the relative importance of the qualitative issues in the ideology of party $p$. If $\pi_p(q) = \pi_p$ for all $q \in \mathbf{Q}$ we have that all qualitative issues have the same effect on the utility of party $p$, thus all qualitative issues are as important in the ideology of party $p$. If $\pi_p(q) > \pi_p(q')$ then issue $q$ is regarded as more important than issue $q'$ by party $p$. Since $\sum_{q \in \mathbf{Q}} \pi_p(q) = 1$, we have that as the value of $\pi_p(q)$ increases, issue $q$ becomes more important for party $p$, and therefore party $p$ requires a more favorable compromise on the other issues for a given deal on $q$.

Similarly, $S_p(\mathbf{s})$ represents the payoff that party $p$ derives from the compromise reached on the quantitative issues. In general one can assume that the payoff that party $p$ derives from the quantitative issues depends on the distribution of executive positions to all coalition members. With this formulation we could represent instances in which a party might care about the difference between the executive positions he obtains in each rank and the positions that some other coalition members obtain in each rank.

However it seems natural to assume that a party only cares about the number of executive positions allocated to himself. In particular, without much loss of generality we could assume that $S_p(\mathbf{s}) = \sum_{s_j \in \mathbf{S}} \mu_p(s_j) \cdot s_j^p$ where $\mu_p(s)$ represents the weight that party $p$ assigns to issue $s$, and it is such that $\mu_p(s) > 0$ and $\sum_{s \in \mathbf{S}} \mu_p(s) = 1$; and since parties derive a larger utility from higher ranked executive positions, we assume that $\mu_p(s_j) > \mu_p(s_{j'})$ for $j < j'$. Finally, $K_p > 0$ represents the relative importance that party $p$ assigns to the quantitative issues with respect to the qualitative issues.

[4] Since the utility function assumed on the qualitative issues is separable, we are assuming that the perception of the parties is that the ideological issues are not interrelated.

Notice that the payoff that party $p$ obtains from the quantitative issues, $S_p(\mathbf{s})$, is always positive while the payoff that he obtains from the qualitative issues, $Q_p(\mathbf{q})$, is always negative. This implies that $K_p S_p(\mathbf{s})$ may be thought of as a reservation value: a party will never accept to become a member of a governing coalition if it has to support a policy compromise on the qualitative issues that gives him a (dis)utility larger than the value that he obtains from the quantitative issues.

This observation allows us to define an Individual Rationality constraint for each party. Formally, the set of policies from which party $p$ derives a utility of zero, $\{(\mathbf{q}, \mathbf{s}) : U_p(\mathbf{q}, \mathbf{s}) = 0\}$, defines the boundary of the set of policies that are Individually Rational for party $p$. The size of this set depends on the magnitude of $S_p(\mathbf{s})$ : the larger the payoff that party $p$ derives from the quantitative issues, the larger the set of policies that party $p$ is willing to support in a given governing coalition. That is, the more a party values to be a member of the governing coalition the more flexible he will be in terms of trading-off policy.

# 3 Negotiation structure and tactics

The negotiation process is initiated with the selection of a formateur: a party that is in charge of making the first offers and it can also be responsible for building up a governing coalition. There is a time limit for the negotiation that is set up by institutional regulations. We assume that this time limit is exogenously given and denoted by $t_{\max}$. After the time limit is reached either we will assume that the game is over or that the games restarts from the beginning. In the first case, the negotiation terminates and if there has not been an agreement within a majoritarian coalition everyone obtains the payoff corresponding to a failure. In the second case, the negotiation process starts again from the beginning.

The negotiation process among the agents consists of a succession of offers and counter offers of values for $(\mathbf{q}, \mathbf{s})$ that continues until an offer is accepted by all the members of a decisive coalition within the maximal time limit or until the time limit is reached. If an offer is accepted by all the members of a decisive coalition within the time limit, the government forms and the policy compromise is implemented. The parties within the governing coalition receive the payoffs corresponding to the implemented policy and all other parties receive a zero payoff. If no decisive coalition reaches an agreement before the time limit, the negotiation is over. In this case we assume that either a new formateur is chosen and the whole process starts again with discounted payoffs or that the game is over and all parties receive a zero payoff.

A sequence of offers and counter-offers is called a negotiation thread. A tactic is a function that generates decision and uses as input a given single criterion. In our case a tactic may generate either an offer in terms of a policy vector, or a decision over which parties to invite to join in a coalition. When generating policy vectors, tactics might be based on criteria such as the amount of time remaining before the maximal time limit of the negotiation, the best offer that a party has received so far, the history of the strategies used by the different parties, expectations on the other parties' behavior, among others. When generating decision over which parties to invite to join in a coalition, tactics might be based on criteria such as properties of the coalition in terms of size, properties of the coalition in terms of the ideologies of the members, etc.

A strategy for a party at a given moment of the negotiation has two main components: the decision over which parties to extend the offer (to invite to join in a coalition) and the kind of offer in terms of a policy choice. We will assume that the strategies of the parties are generated by linear combinations of tactics. The different weights assigned to the different tactics (or criteria) in a given negotiation strategy indicate their relative importance. Since each tactic is based on a specific criterion, the different weights assigned to each tactic represent the relevance or importance assigned to the corresponding criterion. In order to achieve flexibility in negotiation the parties may wish to change their ratings of the different criteria over time. For example, at the beginning of a negotiation thread it may be more important to take into account the competitors' behavior than the remaining time, in which case the tactics that emphasize the behavior of other parties will be given greater weights than those based on the amount of time remaining.

We will now describe different protocols to select the formateur, different procedures that define the negotiation threads, different parties' types when selecting a partner, and different ways to construct a sequence of offers. They correspond to particular examples of tactics that can be used.

## 3.1 Selecting the formateur

We will consider different protocols that select the formateur: the party who starts the negotiation.

Protocol 1: the formateur is the party with the largest share of seats.

Protocol 2: the formateur is chosen by a lottery and each party is selected with a probability proportional to his share of seats.

Protocol 3: the formateur is chosen by a lottery and each party is selected with equal probability.

Formally, the protocol to decide the formateur is given by a lottery $(f_1, ..., f_P)$ such that $f_p \geq 0, \sum_{p \in \mathbf{P}} f_p = 1$ and

Protocol 1: $f_p = 1$ for $p$ such that $v_p > v_{p'}$ for all $p' \neq p$.

Protocol 2: $f_p = v_p$ for all $p$.

Protocol 3: $f_p = \frac{1}{P}$ for all $p$.

We will also consider the possibility of selecting more than one formateur, that is, several parties might be able to start different negotiation threads that would take place simultaneously. In this case the different formateurs may be selected using a random device that, as before, could depend on the parties' seat shares in the parliament.

## 3.2 Choosing a coalition

We may consider three different negotiation procedures for parties to engage in a negotiation: bargaining among parties, bargaining within coalitions, and simultaneous bargaining within coalitions. In each the party that is in charge of engaging the negotiation has to decide which coalition of parties he makes an offer to.

**Bargaining among parties:** The party selected by the protocol becomes the formateur. The formateur has to choose a coalition of parties $C$ and makes an offer to the parties that are members of a coalition $C$.

We will assume an exogenously given ordering of the parties represented by a permutation of the elements of $\mathbf{P}$. The ordering within a given coalition is determined by restricting the application of the ordering on $\mathbf{P}$ to the members of the coalition. We will consider different permutations and analyze their effect on the results. A permutation that orders the parties according to the proportion of seats obtained by each party is of particular relevance in our case.

Parties in $C$ respond sequentially according to the exogenously given ordering by accepting or rejecting the offer. If no party in $C$ rejects the offer then the game is over and the offer is implemented: the coalition $C$ forms the government and the proposed policy is implemented. Otherwise, the first party that rejects the offer becomes the new proposer. He has to choose a coalition of parties $C'$ and make an offer to the parties that are members of coalition $C'$. If no party in $C'$ rejects the offer then the game is over and the offer is implemented: the coalition $C'$ forms the government and the proposed policy is implemented. And so on.

When deciding to whom to make an offer, parties may be of one of three types:

- A party is an **explorer** if he never makes an offer to a party or coalition that made him the last offer.
- A party is a **replier** if he always makes an offer to a party or coalition that made him the last offer.
- A party is of the **mixed-type**, if he makes an offer a party that made him the last offer only if the last offer that he obtained is good enough, otherwise he makes the offer to a different party.

We say that a party considers that an offer is good enough if the utility level that he obtains with that offer is close enough to the utility level that he would obtain with the counter offer he would make. Let $\psi_p \in \Re^+$ denote the threshold used by the party to determine whether the last offer he obtained is good enough.

**Bargaining within coalitions:** The party selected by the protocol becomes the formateur. The formateur has to choose a coalition of parties $C$ and makes an offer to the parties that are members of a coalition $C$.

Parties in $C$ respond sequentially according to the ordering exogenously given, as described before, by accepting the offer, making counter-offers or rejecting the offer. As long as all parties are either accepting offers or making counter-offers the negotiation proceeds. When all parties accept a given offer the negotiation ends, the coalition becomes the governing coalition and the set of policies proposed in the accepted offer are implemented. If a party rejects a coalition (this implies that he rejects the offers that he has received and he chooses not to make a counter-offer) the formateur has to choose a new coalition and start a new negotiation thread.

**Simultaneous bargaining within coalitions:** In this case we assume that at the first stage of the game several formateurs are selected. Each formateur will behave as in the case of bargaining within coalitions. Thus, the negotiation threads described above will proceed in parallel, and a given party may be involved in different negotiation threads at the same point in time. The first negotiation thread that ends successfully is the one that forms the governing coalition.

We can also assume additional properties on the type of coalitions that are called during the negotiation process derived from the existing theories of coalition formation. In particular, parties could consider coalitions based on policy blind theories. In this line we find the Minimal Winning Coalition Theory by von Neumann and Morgenstern (1953), the Minimum Winning Coalition Theory by Riker (1962), and Leiserson's (1966) refinement of the Minimal Winning Coalition Theory using the 'smallest number of parties' bargaining principle. We could also consider coalitions based on theories that assume that policy choice plays a role in the parties' payoffs, such as the Minimal Connected Winning Coalition Theory by Axelrod (1970) and its refinement based on the smallest ideological range by de Swaan (1973).

In addition, we could either restrict parties to negotiate only over minimal winning coalitions or we could allow them to form surplus coalitions. We would need a much more complex framework in order to allow for minority coalitions to form the government.

Finally, we could use the same model and procedure to analyze the effect of the requirement of a q- rule with q $> 1/2$ (supermajorites) over the vote of the parliament in order to form a government.

## 3.3 Offers and counteroffers

An offer from party $p$ at time $t$ is represented by $O_p^t$ where $t$ is an integer that denotes the time at which the proposal is offered, $p \in \mathbf{P}$ denotes the party that offers the proposal. The offer $O_p^t$ denotes a particular value of the vector $(\mathbf{q}, \mathbf{s})$ described before and it represents the policy proposal that this party makes.

When a party receives an offer it has to evaluate it and decide whether to accept it or reject it. If the utility that the party recipient of the offer, $p'$, derives from an offer $O_p^t$ proposed to him is larger than the utility that he would derive from the counter offer that he is ready to send, $O_{p'}^{t+1}$, then party $p'$ accepts the offer. Otherwise he rejects it. After a party has rejected an offer he becomes the new proposer, and therefore, he chooses a coalition and a policy and starts a new negotiation thread.

Formally, for $t \leq t_{\max}$

$$I_{p'}(t, p, p', O_p^t) = \begin{cases} reject & if\ U_{p'}\left(O_p^t\right) < U_{p'}\left(O_{p'}^{t+1}\right) \\ accept & if\ U_{p'}\left(O_p^t\right) \geq U_{p'}\left(O_{p'}^{t+1}\right) \end{cases}$$

For each party we construct a sequence of offers, one for each period $t$, that will be used by the party only when either it receives an offer or it is selected to be a proposer. It is natural to assume that the offers that a party sends out are more and more generous for the party's competitors over time, as the party becomes more and more impatient to reach an agreement. Therefore we assume that the offers send out by a party over time are such that the utility levels obtained by the party from his own offers are declining, starting at an exogenously given initial utility level ($u^0$) at time $t = 0$ until they reach his reservation value as $t$ approaches $t_{\max}$.

An exogenously given functional form will be used to construct the sequence of offers for each one of the parties. Specific features and parameters values of the functional form used will be used in order to indicate the rate of patience or impatience of a party, the speed at which a party is willing to concede, etc... We will use two different families of functions that exhibit this type of behavior: polynomial and exponential

In the polynomial case a functional form that computes the decreasing level of utility over time is given by $u^t = u^0 - u^0 \left(\frac{t}{t_{\max}}\right)^{\frac{1}{\beta}}$ where $\beta > 0$ is a parameter that determines its degree of convexity. When $\beta < 1$ (concave function) the level of utility goes rapidly close to the reservation value and then keeps conceding slowly. These tactics are called 'opening up'. When $\beta = 1$ (linear function) the level of utility moves linearly with time to the reservation value. These are called linear tactics. Finally, when $\beta > 1$ (convex function) the initial level of utility is maintained until the time is almost exhausted, and then it decreases rapidly. These tactics are called 'holding back'. See figure 1.

Similarly, in the exponential case a functional form that computes the decreasing level of utility over time is given by $u^t = 1 + u^0 - \exp\left\{\left(\frac{t}{t_{\max}}\right)^{\beta} \ln\left(1 + u^0\right)\right\}$, where $\beta > 0$ is a parameter that determines its degree of convexity. When $\beta < 1$ (concave function) the level of utility goes rapidly to close to the reservation value. These tactics are called 'opening up'. When $\beta > 1$ (convex function) the initial level of utility is decreases very slowly until the time is almost exhausted, and then it decreases rapidly. These tactics are called 'holding back'. When $\beta = 1$ (convex function) the tactics are 'holding back' but the level of utility moves a bit slower than with higher values of $\beta$. See figure 2.

Comparing the two families of functions we have that on the one hand in both cases values of $\beta$ larger than 1 imply 'holding back' tactics, and values of $\beta$ smaller than 1 imply 'opening up' tactics. On the other hand, for values of $\beta$ larger than 1 the exponential function concedes faster at the beginning than the polynomial one, and for values of $\beta$ smaller than 1 the polynomial function concedes faster at the beginning than the exponential one.

These two families of functions provide a infinite set of tactics (for all possible values of $\beta$) and thus they will allow us to model concession in very different ways.

# 4 Automated negotiation model

Artificial Intelligence's objective when applied to the negotiation framework is to present a formal model with an automated and tractable negotiation mechanism for autonomous agents, although the outcomes might not be optimal. A multi-agent system (MAS) is a system composed of multiple interacting intelligent agents. The agents in a multi-agent system have several important characteristics:

- Autonomy: the agents are at least partially autonomous
- Local views: no agent has a full global view of the system, or the system is too complex for an agent to make practical use of such knowledge
- Decentralization: there is no one controlling agent (or the system is effectively reduced to a monolithic system)

Typically multi-agent systems research refers to software agents. However, the agents in a multi-agent system could equally well be robots, humans or human teams. A multi-agent system may contain combined human-agent teams. Multi-agent systems can manifest self-organization and complex behaviors even when the individual strategies of all their agents are simple.

In the AI literature we can find different examples of protocols for many-to-many negotiations. In Kraus, Wilkenfeld and Zlotkin (1995) the authors introduce an strategic model of negotiation that takes the passage of time during the negotiation process into account. A distributed negotiation mechanism is introduced that is simple, efficient and stable. Using this negotiation mechanism autonomous agents have strategies that result in efficient agreements without delays. In their model they consider the problem where agreements involve all the agents, but they don't deal with situations in which agents are free to form any coalition that includes some of the agents while excluding others. In Dang and Huhns (2005) and in Nguyen and Jennings (2004) the authors introduce two approaches that differ from ours because they consider concurrent negotiations that are either multiple one-to-many or many-to-many bilateral.

In the present section we sketch an algorithm for a many-to-many multilateral negotiation protocol. We assume that the delivery time is negligible comparing to the time interval of each negotiation round. First of all, for the sake of clarity we introduce the following distinctions among sets of parties, in relation with their autorization to start negotiations, their desire to continue a negotiation thread or the fact that they have sent a message to a certain agent.

$A_p^t$ as the set of parties with whom party $p$ is authorized to lead a negotiation at time $t$
$S_p^t$ as the set of parties with whom party $p$ wants to negotiate at time $t$
$Q_p^t$ as the set of parties from whom party $p$ receives messages at time $t$

The agents communicate and compromise to reach mutually beneficial agreements. We will use the following notation for representing the negotiation messages:
$O_{p\to q}^t$ as the proposal that party $p$ offers to party $q$ at time $t$
$M_{p\to q}^t$ as the message that party $p$ sends to party $q$ at time $t$
$M_{p\to q}^t \in \{O_{p\to q}^t, Accept, Reject, Pre-Accept, Over\}$

Table 1: Negotiation Messages

| | |
|---|---|
| Accept | An agent formalizes the pre-accepted offer ending the negotiation |
| Pre-Accept | An agent pre-accepts a previous offer |
| Reject | An agent rejects a previous offer |
| Over | An agent ends a negotiation thread |

Automated negotiation is a key form of interaction in systems that are composed of multiple autonomous agents. The objective of these interactions is to reach an agreement through an iterative process of making offers. The content of the proposals is a function of the strategy of the agents. The sketch of algorithm we present here enables software agents to generate offers during the negotiation. In the formalization of the algorithm we distinguish, at any given time $t$, two types of agents, one agent that leads the negotiation at time $t$, the formateur (that we identify with the property $A_p^t \neq \emptyset$) and the rest of agents that either answer their proposals or remain silent, because the formateur do not negotiatie with them.

A Reject Message has as a consequence that a new negotiation thread starts, involving new agents, and according to the protocol chosen, a new leader of the negotiation taking the responsibility of getting to an agreement. An Accept Message finishes the negotiation while a Pre-Accept Message is a provisional acceptance of an offer, submitted to further negotiation if one of the agent does not pre-accept the offer made by the formateur. Since there is an institutional maximum time limit, the protocol has the termination property, that is, guarantees that any negotiation process following it will eventually terminate. For the sake of clarity we don't include here the instructions for time $t = t_{max}$.

**Initialization**

**if** agent $p$ is a formateur (that is, $A_p^{t_0} \neq \emptyset$) **then**

**sends** $O_{p\to q}^{t_0}$ to all $q \in S_p^{t_0} \cap A_p^{t_0}$

**Negotiation**

**while** $t < t_{max}$ **do**

**if** $A_p^{t+1} \neq \emptyset$ **then**

in case that, for some $q_0 \in Q_p^t$, $q_0 \notin S_p^{t+1}$

**sends** *Reject* to every $q \in A_p^{t+1} \cap Q_p^t$

in case that, for some $q_0 \in Q_p^t$, $M_{q_0 \to p}^t = Reject$

**sends** *Over* to every $q \in S_p^{t+1} \cap A_p^{t+1}$

otherwise

in case that, for every $q_0 \in Q_p^t$, $M_{q_0 \to p}^t = Pre - Accept$

**sends** *Accept* to every $q \in S_p^{t+1} \cap A_p^{t+1}$ and **then End.**

in case that, for some $q_0 \in Q_p^t$, $M_{q_0 \to p}^t = O_{q_0 \to p}^t$

**if** $U(O_{q_0 \to p}^t) \geq \max(U(O_{p \to q_0}^{t+1}), \max\{U(O_{r \to p}^t) : r \in S_p^{t+1} \cap Q_p^t\}$

**sends** $Pre - Accept(O_{q_0 \to p}^t)$ to $q_0$

**and** for every $q \in S_p^{t+1} \cap A_p^{t+1}, q \neq q_0$, **sends** $O_{p \to q}^{t+1} = O_{q_0 \to p}^t$

**otherwise send** $O_{p \to q}^{t+1}$ to every $q \in S_p^{t+1} \cap A_p^{t+1}$

in case that, for some $q_0 \in Q_p^t$, $M_{q_0 \to p}^t = Over$

**sends** $O_{p \to q}^{t+1}$ to all $q \in S_p^{t+1} \cap A_p^{t+1}$

**if** $A_p^{t+1} = \emptyset$ **then**

for every $q \in Q_p^t$ but $q \notin S_p^{t+1}$ **sends** *Reject*

and for every $q \in S_p^{t+1} \cap Q_p^t$ **then**

**case** $M_{q \to p}^t = O_{q \to p}^t$ **then**

**if** $U(O_{q \to p}^t) \geq U(O_{p \to q}^{t+1})$

**send** $Pre - Accept(O_{q \to p}^t)$

**otherwise** **send** $O^{t+1}_{p\to q}$

**case** $M^t_{q\to p}$ = *Over* **then** don't send any message.

**case** $M^t_{q\to p}$ = *Reject* **then** don't send any message.

**case** $M^t_{q\to p}$ = $Pre - Accept$ **then** **sends** $Pre - Accept$

# 5 Negotiation strategies

We will analyze the outcomes obtained with different negotiation strategies, that is, different linear combinations of tactics for different and relevant values of the parameters.

The simplest set up would be obtained if we assume that a negotiation strategy consists of a single tactic that is kept constant over time, that is, all weights of the linear combination are equal to zero except for one. Increasing the sophistication level we may consider that the weights of the linear combination are constant.

A more complex set up would have these weights changing according to the history of the negotiation, that is, according to the results obtained in the previous negotiation rounds. Finally, we could reach a higher level of sophistication by considering that the weights change according to the expectations that the parties build upon the behavior of their competitors as the negotiation process evolves.

We want to evaluate the performance of the strategies according to: the utility obtained by the parties, the number of deals made, and the net payoffs obtained by the parties computed as costs the number of negotiations rounds needed to obtain a deal. When analyzing the effect of the different protocols to select the formateur we have to compare the three protocols defined above for different values of the parameter $v_p$ representing the vote share. Regarding the three types of behavior assumed when bargaining among parties,we need to compare the effect of each one being used against any combination of types of the competitors and in addition we will have to consider the effect induced by assuming different values of the threshold $\psi_p$.

Finally, we have to analyze the effect of the different ways considered to construct an offer. In this case we have to distinguish between: a polynomial versus an exponential function; 'holding back' versus 'opening up' tactics, for different values of the parameter $\beta$; different values of the initial level of utility $u^0$, and different values of the time limit $t_{\max}$. And we also have to analyze the effect of each one of these tactics when played against any combination of types for the competitors. In addition we will have to consider the effect of different values for the parameters of the payoff functions of the parties such as: the parties' ideal points and weights on the qualitative issues, the parties' weights on the quantitative issues, and the relative weight that parties' assign to quantitative issues with respect to qualitative issues.

From the combination of the formal model of government formation and the automated negotiation protocol described before, we should be able to test some hypotheses based on expected results. We list some of them here:

1. **Protocols:** a larger probability of being selected as a formateur, $f_p$, implies a clear advantage in terms of a larger probability of making a deal and a larger utility.

2. **Types of partners:** when most parties are repliers the formateur has an advantage. In particular, if there is a deal, he is always in it independently of his type. The number of possible coalitions increases with the proportion of explorers, and when all parties are explorers any coalitions is possible. Large values of $\psi_p$ imply a behavior replier-like and small values of $\psi_p$ imply a behavior explorer-like. Thus the effect of different values for this parameter should follow from the ones described above for the different types. Figures 3 to 8 illustrate the results for the case of three parties.

3. **Types of tactics:** the 'holding back' tactics imply:

   • smaller number of deals, and this effect is worse in the polynomial case when $t_{\max}$ is small.

   • a larger utility, given that there is a deal.

   • smaller number of deals and larger utility when $t_{\max}$ is large

   • small number of deals

   'opening up' tactics imply:

   • smaller utility and this effect is worse in the exponential case when $t_{\max}$ is large.

   • larger utility for small $t_{\max}$ and smaller utility for large $t_{\max}$

   • larger number of deals for small $t_{\max}$

   • smallest utility given a deal

4. **Maximal time:** larger $t_{\max}$ implies a larger number of offers, therefore 'opening up' tactics imply larger net payoffs for small $t_{\max}$.

Most of the empirical work on the politics of coalition in parliamentary democracies seek to account for the coalitions that actually form. A comprehensive survey can be found in Laver and Schofield (1990). Martin and Stevenson (2001) provide a list of the properties mostly observed in governing coalitions such as: evidence of minimal winning coalitions forming as opposed to surplus or minority coalitions, coalitions with fewer number of parties, coalitions that contain the party with the largest proportion of seat, coalitions with smaller ideological divisions are most likely. However, Laver and Schofield (1990) show that most governments are either minority or surplus governments. Finally, Diermeier and Merlo (2004) show that in most cases the largest party is not selected as the formateur. We expect that the results obtained from our proposal would offer new explanations to the existing empirical findings, and would shed some light on the contradicting ones.

# 6 Concluding remarks

The combination of a formal model of government formation with an automated negotiation mechanism for autonomous agents described in this paper should provide some new insights on how to develop tractable formal models of government formation that could help us to understand how a given election result leads to a given government. Furthermore, the results that could be obtained from this combination might be the source of new explanations to some of the existing empirical findings. These are the two main academic goals of this project in the subfield of government formation.

In addition this project will produce a contribution to the literature of artificial intelligence, since the algorithm for a many-to-many multilateral negotiation protocol extends the existing automated negotiation models mostly based on either bilateral or one-to-many negotiations.

Finally, the ultimate goal of this project is to provide some recommendations regarding bargaining behavior to agents that engage in real bargaining for government formation situations. We aim to obtain an evaluation of the performance of a large class of bargaining strategies. Having a characterization of the performance of a given strategy in a number of qualitative different environments, would allow for specific recommendations of given strategies on particular environments.

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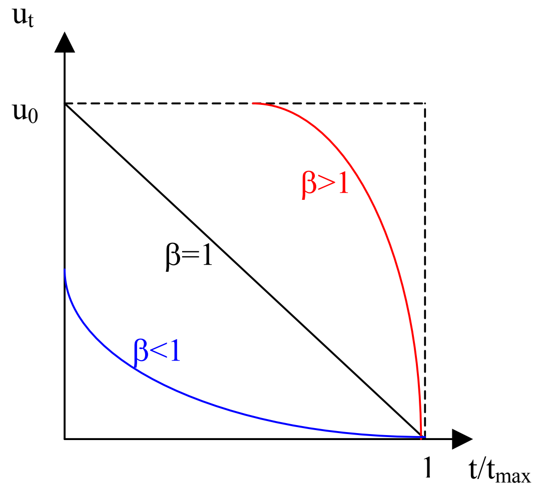



**Figure 1:** Polynomial case: β>1 represents a 'holding back' strategy, β<1 represents an 'opening up' strategy, and β=1 represents a linear strategy.

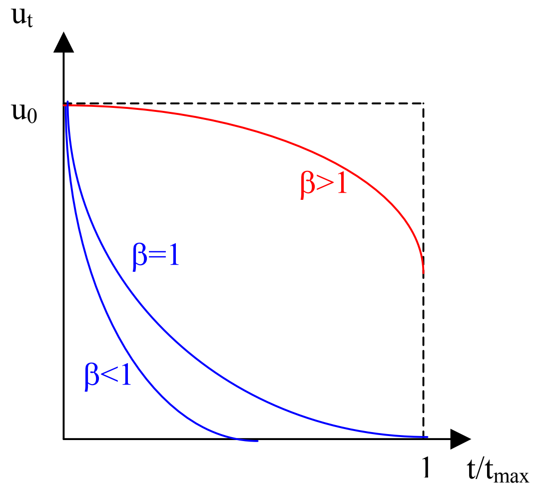



**Figure 2:** Exponential case: β>1 represents a 'holding back' strategy and β<1 represents an 'opening up' strategy.

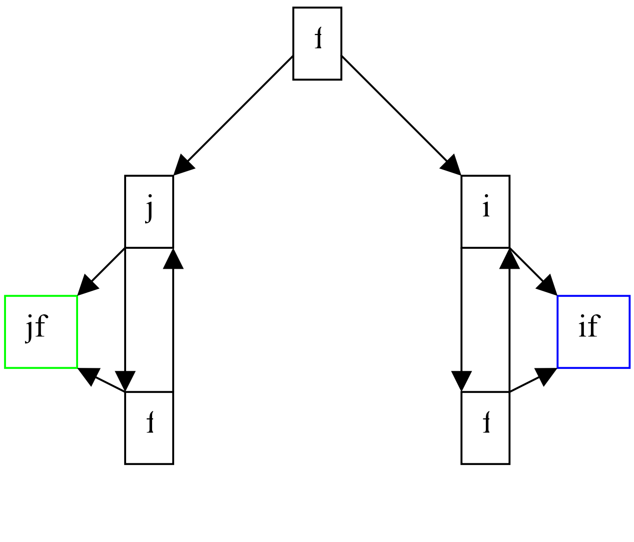



**Figure 3:** All parties are REPLIERS.

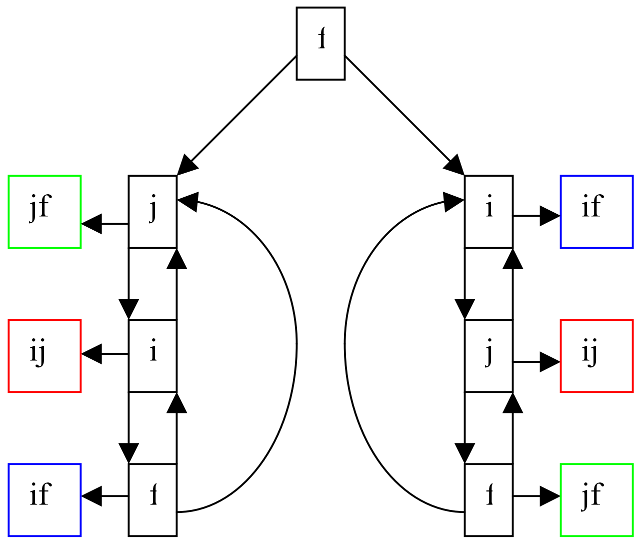



**Figure 4:** All parties are EXPLORERS.

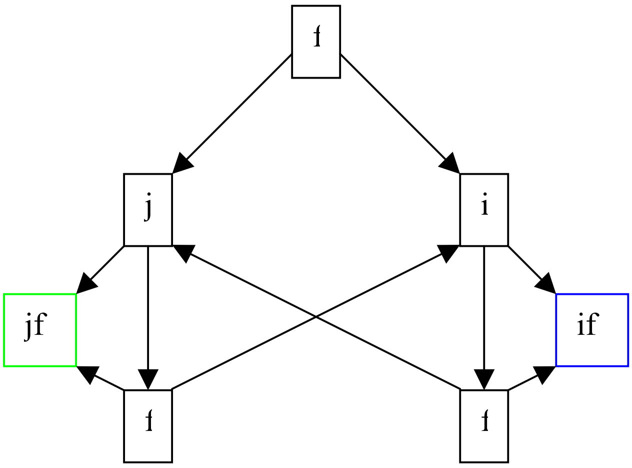



**Figure 5:** The formateur (f) is an EXPLORER and the others (i and j) are REPLIERS.

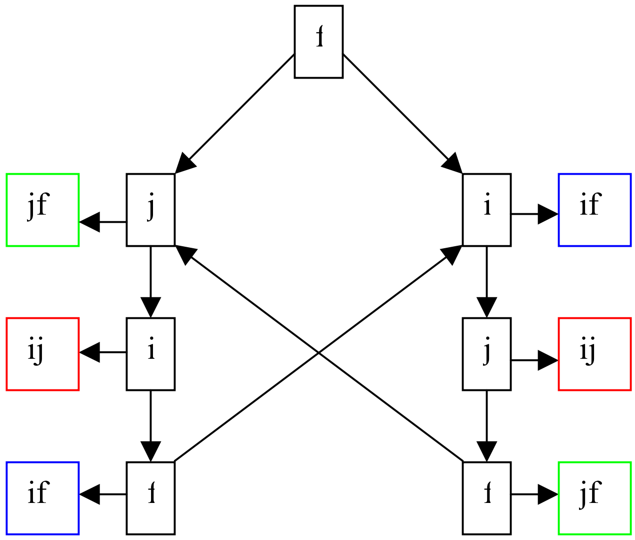



**Figure 6:** The formateur (f) is a REPLIER and the others (i and j) are EXPLORERS.

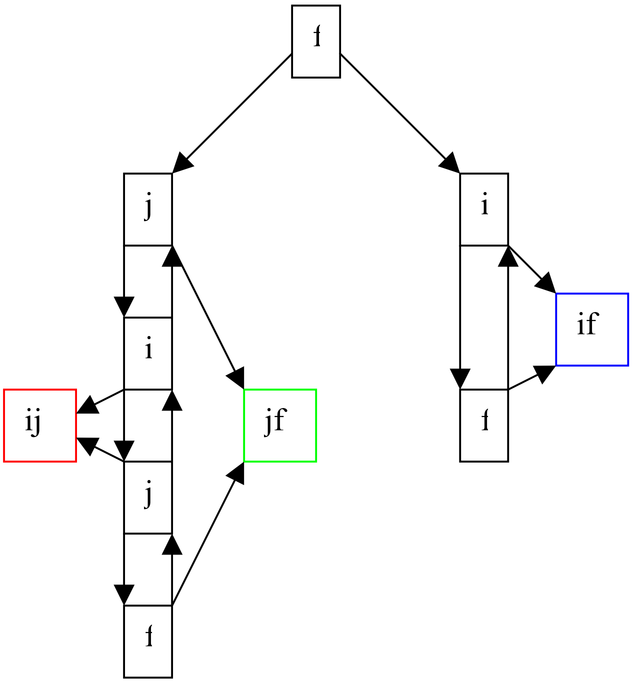



**Figure 7:** The formateur (f) and one of the parties (i) are REPLIERS and the other party (j) is an EXPLORER.

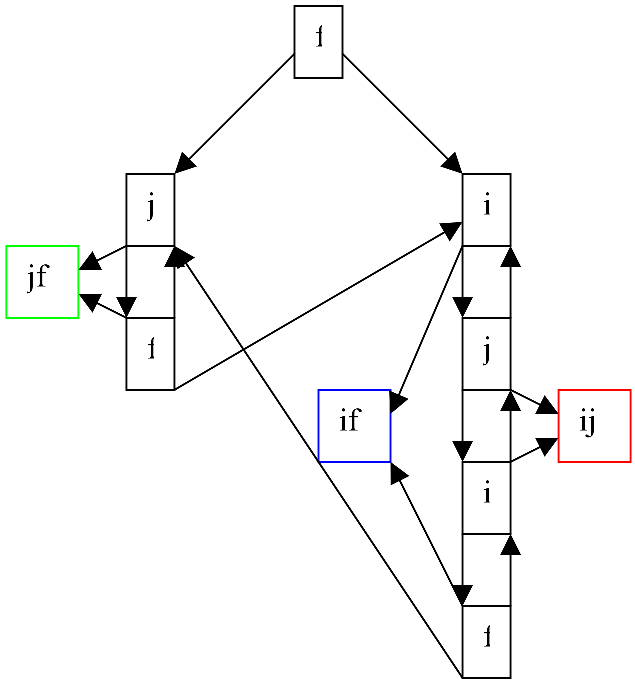
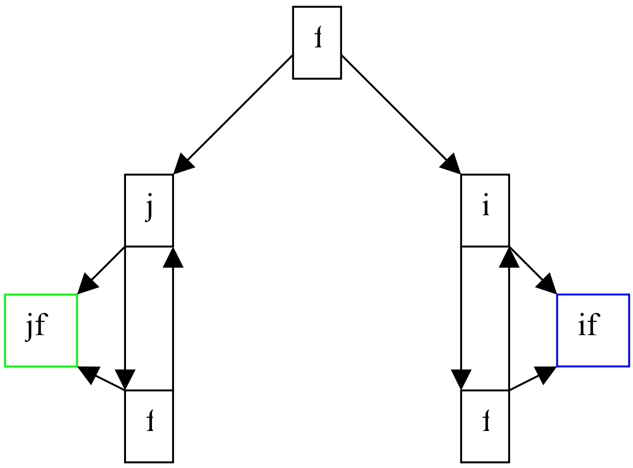
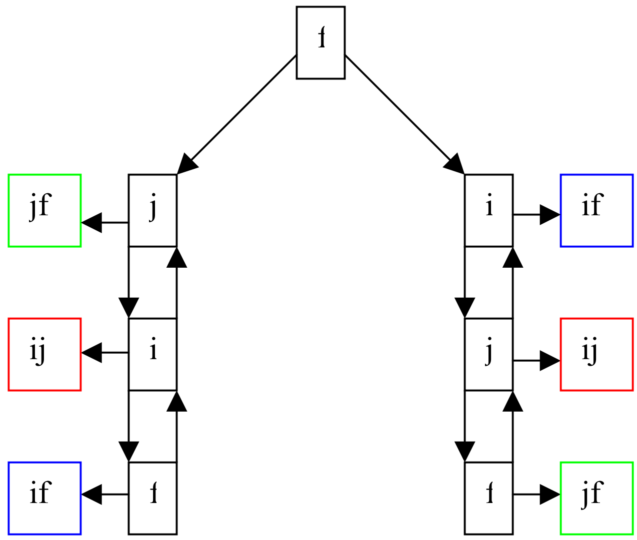



**Figure 8:** The formateur (f) and one of the parties (i) are EXPLORERS and the other party (j) is a REPLIER.




REPLIERS




EXPLORERS